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الفصل السادس قسم الاتصالات

Electromagnetic 2

Homework

Q 1 Let $H_s = (2\angle{-40^\circ}\,\vec{a_x} - 3\angle{20^\circ}\,\vec{a_y})e^{-j0.07z}$ A/m for a uniform plane wave traveling in free space find :-
a) ω , b) H_x at $p(1,2,3)$ at $t = 31ns$, c) $|H|$ at $t=0$ at the origin

a) $H_s = (2\angle{-40^\circ}\,\vec{a_x} - 3\angle{20^\circ}\,\vec{a_y})e^{-j\beta z}$ so $\beta = 0.07$ rad/m
∴ The wave is Traveling in free space $\Rightarrow \beta = \frac{\omega}{c} \Rightarrow \omega = \beta c$
∴ $\omega = 0.07 \times 3\times10^8 = 21\times10^6$ rad/s $= 21$ Mrad/s

b)

Q 2 let us apply these results to a 1MHz plane wave propagating in fresh water , At this frequency losses in water are known to be small , so for simplicity we will neglect ϵ'' . (بمعنى أن إهمالها يعني أن $\sigma = 0$)
In water $\mu_r = 1$ and at 1MHz $\epsilon'_r = \epsilon_r = 81$
(β , λ , V_p , η , E , H , مطاليب السؤال)

$\omega = 2\pi f = 2\pi\times10^6$ rad/s , $\mu = \mu_0$, $\epsilon' = 81\epsilon_0$

$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r} = \frac{\omega}{c}\sqrt{(1)(81)} = \frac{2\pi\times10^6}{3\times10^8}\times 9 = \frac{3}{50}\pi = 0.1884$ rad/m

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\frac{3\pi}{50}} = \frac{100}{3} = 33.33$ m , $V_p = \frac{\omega}{\beta} = \frac{2\pi\times10^6}{\frac{3\pi}{50}} = 33.\times10^6$ m/s
33.33×10^6 m/s

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{81}} = \frac{40}{3}\pi = 41.8879\ \Omega$$

$$E = E_0 \cos(\omega t - \beta z)\ \vec{a_x}\ V/m$$
$$= E_0 \cos(2\pi \times 10^6 t - 0.188 z)\ \vec{a_x}\ V/m$$

$$H = H_0 \cos(\omega t - \beta z)\ \vec{a_y}\ A/m \implies H_0 = \frac{E_0}{\eta} = \frac{E_0}{41.88}$$
$$= \frac{E_0}{41.88} \cos(2\pi \times 10^6 - 0.188 z)\ \vec{a_y}\ A/m$$

يمكن نفرض أي قيم لـ E_0 وبالتالي نجاد E ، H

Q3 if $\eta_1 = 100\ \Omega$ ، $\eta_2 = 300\ \Omega$ ، $E^+_{x10} = 100\ V/m$

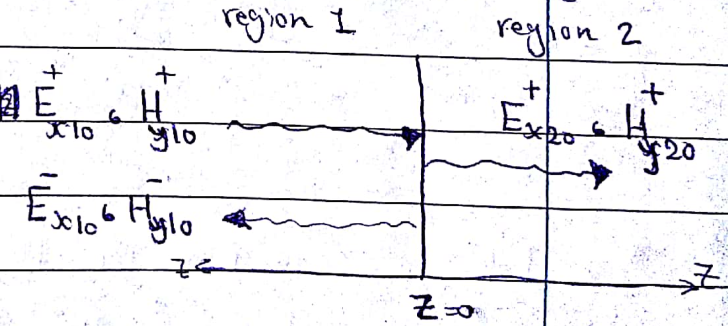
calculate values for the incident ، reflected ، transmited waves

NoTe:- E_x تعني $\to E\vec{a_x}$ ، E^+_1 تعني الموجة الساقطة في المنطقة I ، E_0 تعني أقصى قيمة للموجة

so $E^+_{x10} \to$ تعني أقصى قيمة للموجة الساقطة في المنطقة I اتجاه تقدمه $\vec{a_x}$

+ ساقطة

- منعكسة



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{200}{400} = 0.5$$

$$\tau = 1 + \Gamma = 1 + 0.5 = \frac{3}{2}$$

$$\Gamma = \frac{E^-_{x10}}{E^+_{x10}} \to E^-_{x10} = 0.5\ E^+_{x10} = 0.5 \times 100 = 50\ V/m = -50\ V/m$$

الإشارة السالبة تدل على أن الموجة عكس الاتجاه

$$\tau = \frac{E^+_{x20}}{E^+_{x10}} \to E^+_{x20} = \tau\ E^+_{x10} = 1.5 \times 100 = 150\ V/m$$

$$H^+_{y10} = \frac{E^+_{x10}}{\eta_1} = \frac{100\ V/m}{100\ \Omega} = 1\ A/m \ ،\ H^-_{y10} = \frac{E^-_{x10}}{\eta_1} = \frac{-50}{100} = -0.5\ A/m$$

$$H^+_{y20} = \frac{E^+_{x20}}{\eta_2} = \frac{150}{300} = 0.5\ A/m$$

يمكن إيجاد موجة القدرة من خلال

$$P = \frac{1}{2}EH \ ،\ P^+_1 = \frac{1}{2}E^+_{x10}H^+_{y10} = 100\ w/m^2$$

$$P^-_1 = \frac{1}{2}E^-_{x10}H^-_{y10} = 25\ w/m^2$$

$$P^+_2 = \frac{1}{2}E^+_{x20}H^+_{x20} = 75\ w/m^2$$

Q4 A 1 MHz uniform plane wave is normally incident onto a fresh water lake ($\varepsilon'_r = 78$, $\varepsilon''_r = 0$, $\mu_r = 1$). Determine the fraction of the incedent power that is :-
a) reflected, b) transmitted, c) determine the electric feild that is transmitted into the lake.

$\varepsilon = 78\varepsilon_0$, $\mu = \mu_0$, $\omega = 2\pi f = 2\pi \times 10^6$ rad/s

$\varepsilon''_r = 0 \rightarrow \varepsilon''_r = \frac{\sigma}{\omega} = 0$ so $\sigma \simeq 0$

$B_1 = \frac{\omega}{c} = \frac{2\pi \times 10^6}{3\times 10^8} = \frac{\pi}{150}$, $\eta_1 = 377\Omega$

$B_2 = \frac{\omega}{c}\sqrt{78} = 0.1849$ rad/m, $\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{1}{78}} = 42.6858\Omega$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta} = -0.796$

a) the fraction of the incident power that is reflected

$$\frac{P_r}{P_{inc}} = |\Gamma|^2 = |-0.796|^2 = 0.6336$$

b) the fraction of the incident power that is transmitted

$$\frac{P_t}{P_{inc}} = 1 - |\Gamma|^2 = 1 - |-0.796|^2 = 0.3663$$

c) $T = 1 - \Gamma = 1 - 0.796 = 0.204$ Let put $E^+_{inco} = 1$ V/m

so $\rightarrow T = \frac{E^+_{to}}{E^+_{inco}} \rightarrow E^+_{to} = T E^+_{ino} = 0.204 \times 1$ V/m

so $E^+_t = 0.204$ V/m

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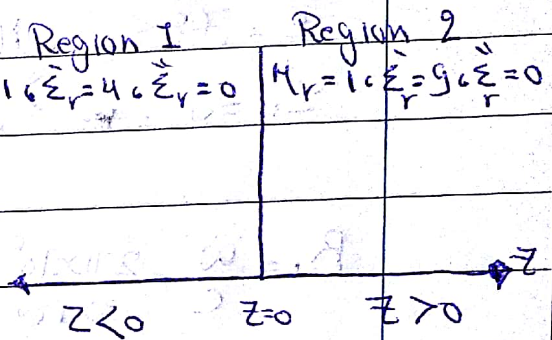
Q5 To illustrate some of these results, let us consider a 100 V/m, 3 GHz wave that is propagating in material having ($\varepsilon'_{r_1} = 4$, $\mu_{r_1} = 1$, $\varepsilon''_r = 0$) the wave is normally incident on another perfect dielectric in region 2 ($z > 0$), where ($\varepsilon'_{r_2} = 9$, $\mu_{r_2} = 1$) we seek the locations of the Max and min of E

$f = 3\,GHz$, $E^{+}_{xs1} = 100\, e^{-j\beta z}$



$\omega = 2\pi f = 2\pi \times 3\times10^{9} = 6\pi\times10^{9}$ rad/s $= \omega_1 = \omega_2$

region 1 $\beta_1 = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{4}$

or $\beta_1 = \frac{6\pi\times10^{9}}{3\times10^{8}} \times 2 = 40\pi = 125.663$ rad/m

$\lambda_1 = \frac{2\pi}{\beta} = \frac{2\pi}{40\pi} = \frac{1}{20} = 0.05\,m = 5\,cm$, $\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{1}{4}} = 60\pi\,\Omega$

region 2 $\beta_2 = \frac{\omega}{c}\sqrt{9} = \frac{6\pi\times10^{9}}{3\times10^{8}} \times 3 = 60\pi$ rad/m, $\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{60\pi} = 0.033\,m$ $3.33\,cm$

$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{9}} = 40\pi\,\Omega$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 60\pi}{40\pi + 60\pi} = -0.2$ $\Gamma < 0$ $\eta_2 < \eta_1$

$E^{-}_{xs1} = \Gamma E^{+}_{xs1} = 20\, e^{-j\beta z} = -20\, e^{j40\pi z}$

so The minimum E_{min} $E^{+}_{xs1} - E^{-}_{xs1} = 80\, e^{-j40\pi z} = 80$ V/m $|E|_{min}$ → Region 1

The maximum E_{max} $E^{+}_{xs1} + E^{-}_{xs1} = 120\, e^{-j40\pi z} = 120$ V/m $|E|_{max}$

There is no reflected waves in Region 2

Q6 A uniform plane wave is incident from air onto glass at an angle of 30°. Determine the fraction of the incident power that is reflected and transmitted for:- a) p-polarization, b) s-polarization. Glass has reflected index $n_2 = 1.45$

$\theta_1 = \theta_i = 30°$, $n_1 = 1$ (air) , $n_2 = 1.45$ (glass) , $\theta_2 = ? = \theta_t$

تسمى زاوية الانكسار أو الانتقال

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_t = \theta_2 = \sin^{-1}\left(\frac{\sin\theta_i}{n_2}\right) = \sin^{-1}\left(\frac{\sin 30}{1.45}\right) = 20.2°$

$\eta_1 = 120\pi = 377\,\Omega$ (air) , $\eta_2 = \frac{\eta_1}{n_2} = \frac{377}{1.45} \simeq 260\,\Omega$

$\eta_{1p} = \eta_1 \cos\theta_i = 326\,\Omega$, $\eta_{2p} = \eta_2 \cos\theta_t = 244\,\Omega$

for p: $\Gamma_p = \frac{\eta_{2p} - \eta_{1p}}{\eta_{2p} + \eta_{1p}}$ or $\Gamma_p = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = -0.1439$

a) fraction of incident power that is reflected

$\frac{P_r}{P_{inc}} = |\Gamma_p|^2 = 0.207$

- fraction of incident power that is transmitted

$\frac{P_t}{P_{inc}} = 1 - |\Gamma_p|^2 = ~~0.957~~ = 0.9792$

for s: $\Gamma_s = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} = -0.2215$ [$\theta_t = 20.2°$, $\theta_i = 30°$]

b) $\frac{P_r}{P_{inc}} = |\Gamma_s|^2 = 0.049$

$\frac{P_r}{P_{inc}} = 1 - |\Gamma_s|^2 = 0.9509$

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The plane $y=0$ defines the boundary between Two diffrent dielectrics. For $y<0$ ($\varepsilon'_{r1}=1$, $\mu_1=\mu_0$, $\varepsilon''_{r1}=0$). For $y>0$ ($\varepsilon'_{r2}=5$, $\mu_2=\mu_0$, $\varepsilon''_{r2}=0$), Let $E^+_{z1}=150\cos(wt-8y)$ V/m

Find :- a) w, b) H^+_1, c) H^-_1

Region 1	Region 2
$\varepsilon'_r=1$, $\varepsilon''_r=0$, $\mu=\mu_0$	$\varepsilon'_r=5$, $\varepsilon''_r=0$, $\mu=\mu_0$
$y<0$	$y>0$

$y=0$ z

Region 1

$\varepsilon=\varepsilon_r\varepsilon_0=\varepsilon_0$, $\mu=\mu_0$ $\boxed{\varepsilon_r=\mu_r=1}$

(air) منها يمكن القول بأن هذه المنطقة هي

$E^+_{z1}=150\cos(wt-8y)$ V/m

$B=8$ rad/m $\rightarrow B=\frac{w}{c} \rightarrow w=Bc=8\times3\times10^8=24\times10^8$ rad/s

a) $w=w_1=w_2=24\times10^8$ rad/m, $\eta_1=\eta_0=377\Omega$

b) $E^+_{z1}=E_{z10}\cos(wt-By)\,\vec{a_z}$ V/m

$H^+_{x1}=\frac{E_{z10}}{\eta_1}\cos(wt-By)\,\vec{a_x}$ A/m

$H^+_{x1}=0.397\cos(24\times10^8t-8y)\,\vec{a_x}$ A/m

c) $\eta_2=\sqrt{\frac{\mu}{\varepsilon}}=\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}=120\pi\sqrt{\frac{1}{5}}=168.5955\ \Omega$

$\Gamma=\frac{\eta_2-\eta_1}{\eta_2+\eta}=-0.3819 \rightarrow E^-_{z1}=\Gamma E^+_{z1}=-57\cos(24\times10^8t+8y)\,\vec{a_z}$ V/m (.2964)

$H^-_{x1}=\frac{E^-_{z1}}{\eta_1}\cos(wt-By)\,\vec{a_x}$ A/m $=0.1519\cos(24\times10^8t+8y)\,\vec{a_x}$ A/m

Q8 Consider these regions in which $\varepsilon''=0$, Region 1 $z<0$ ($\mu_1=4\mu$ H/m, $\varepsilon'_1=10$ pF/m), Region 2 $0<z<6$ cm ($\mu_2=2\mu$ H/m, $\varepsilon'_2=25$ pF/m), Region 3 $z>6$ cm ($\mu_3=\mu_1=4\mu$, $\varepsilon'_3=\varepsilon_1=10$ pF/m)

a) what is the lowest frequency at which a uniform plane wave incident from region I onto the boundary at $Z=0$ will have no reflection?

b) If 50 MHZ, what will the standing wave ratio be in Region I?

$B_2 = \omega\sqrt{\mu\varepsilon}$, $d = 6\,cm$

a) $\boxed{B_2 d = \pi}$, $B_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{\mu_2\varepsilon_2}$ ← الشرط

$B_2 = \omega\sqrt{2\times10^{-6}\times25\times10^{-12}} \Rightarrow B_2 = 7.071\times10^{-9}\,\omega$

$7.071\times10^{-9}\times6\times10^{-2}\,\omega = \pi$ بالتعويض في 1

$\omega = \dfrac{\pi}{4.24\times10^{-10}} = 7.409\times10^{9} \rightarrow f = \dfrac{\omega}{2\pi} = \dfrac{7.409\times10^{9}}{2\pi} = 1.179\,GHZ$

b) $\omega = 2\pi\times50\times10^{6}$, $B_2 = \omega\sqrt{\varepsilon_2\mu_2} = 2.22\,rad/m$, $B_2 d = 2.22\times6\times10^{-2} = 0.133$

$\eta_1 = \eta_3 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}} = \sqrt{\dfrac{4\times10^{-6}}{10\times10^{-12}}} = 632\,\Omega$

$\eta_{in} = 283\left[\dfrac{632\cos(0.133) + j283\sin(0.13)}{283\cos(0.133) + j632\sin(0.13)}\right] = 589 - j138 = 605\angle{-23}$

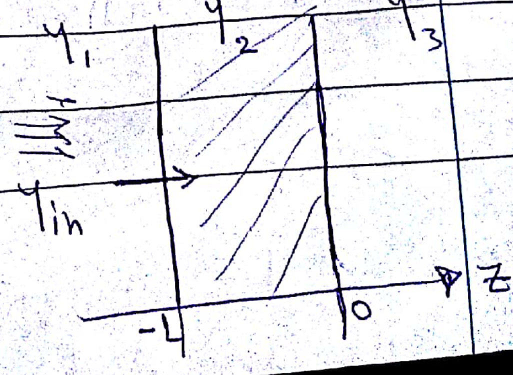
$\Gamma = \dfrac{\eta_{in} - (\eta_1 = \eta_3)}{\eta_{in} + (\eta_1 = \eta_3)} = 0.12\angle{-1.7}$

$SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.12}{1 - 0.12} = 1.2727$

Q9 let $\eta_1 = \eta_3 = 377\,\Omega$ and $\eta_2 = 0.4\eta_1$. A uniform plane wave is normally incident from the left as shown, plot curve of the standing wave ratio, s in the region to the left

a) as function of L if $f = 2.5\,GHZ$

b) as function of f if $L = 2\,cm$



$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$\therefore \beta = \omega\sqrt{\mu\varepsilon} = \frac{n\omega}{c}$

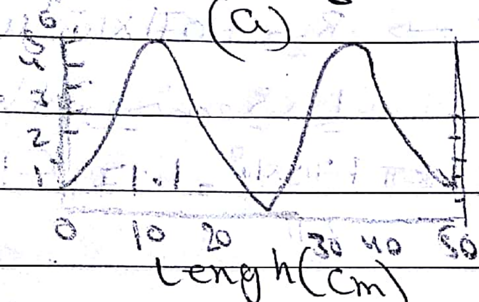
$\therefore \eta_2 = 0.4\,\eta_1 \rightarrow \eta_2 = \frac{\eta_1}{n} \rightarrow n = \frac{1}{0.4} = 2.5 \quad (\mu = \mu_0 ,\ \varepsilon = \varepsilon_0)$

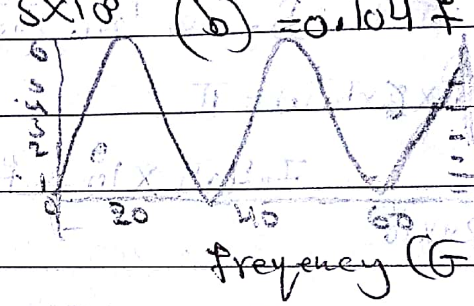
a) $f = 2.5\,GHz$, $\beta l = \frac{n\omega}{c} l = \frac{(2.5)(2\pi \times 2.5 \times 10^9)}{3\times 10^8} l = 12.95\, l$ (l in meter)

$0.1295\, l$ (l in cm)

b) $l = 2\,cm$, $\beta l = \frac{2\pi n l}{c} f = \frac{(2\pi\times 2.5)(2\times 10^{-2})}{3\times 10^8} f = 1.04\times 10^{-10} f$ (f in Hz)

$= 0.104\, f$ (f in GHz)

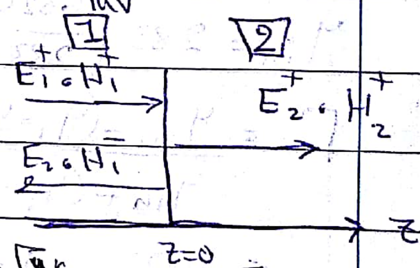




Q10 Let Region 2 be free space while ($\mu_{r_1} = 1$, $\varepsilon''_{r_1} = 0$) ε'_{r_1} is unknown

Find ε'_{r_1} if: a) Amplitude E_1^- half E_1^+ , b) P^-_{1av} is half P^+_{1av} c) $|E_1|_{min}$ is half $|E_1|_{max}$

R_2 free space $\eta_2 = 377\,\Omega$



a) $\Gamma = \frac{E_1^-}{E_1^+}$ $E_1^- = 0.5 E_1^+ \rightarrow \Gamma = \frac{0.5 E_1^+}{E_1^+} = 0.5$

$\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 377\sqrt{\frac{1}{\varepsilon_r}}$

$\sqrt{\varepsilon_r} = \frac{377}{\eta_1} \rightarrow \eta_1 = \frac{377}{\sqrt{\varepsilon_r}}$ $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{377 - \frac{377}{\sqrt{\varepsilon_r}}}{377 + \frac{377}{\sqrt{\varepsilon_r}}} = 0.5 \rightarrow \varepsilon_{r_1} = 9$

b) $\Gamma = \frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \rightarrow \frac{P_1^-}{P_1^+} = |\Gamma|^2 = 0.25 \rightarrow \varepsilon_r = 34$

c) $\frac{|E_1|_{max}}{|E_1|_{min}} = S' = \frac{1 + |\Gamma|}{1 + |\Gamma|} = 2 \rightarrow \Gamma = \frac{1}{3} = \frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \rightarrow \varepsilon_r = 4$

Electromagnetic 2 الإسم:- أحمد فهمي محمد بلاغة

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Home work 2 الفصل السادس / قسم الاتصالات

Q_1 a) Given a non magnetic material having $\varepsilon'_r = 3.2$ and $\sigma = 1.5 \times 10^{-4}$ S/m, find numerical values at 3 MHz for the (a) loss tangent, b) attenuation constant, c) phase constant, d) intrinsic impedance.

$f = 3$ MHz, $\omega = 2\pi \times 3 \times 10^6$, $\varepsilon'_r = \varepsilon_r = 3.2$, $\sigma = 1.5 \times 10^{-4}$ S/m

Non-magnetic $\xrightarrow{so}$ $\mu_r = 1$

a) loss tangent $\rightarrow \dfrac{\sigma}{\omega\varepsilon} = \dfrac{1.5 \times 10^{-4}}{2\pi \times 3 \times 10^6 \times 3.2 \times \frac{10^{-9}}{36\pi}} = \dfrac{9}{32} = 0.28125$

b) attenuation constant $\rightarrow \alpha = \dfrac{\omega}{c}\sqrt{\dfrac{\mu_r \varepsilon_r}{2}}\sqrt{\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} - 1}$ $\boxed{\dfrac{\sigma}{\omega\varepsilon} = \dfrac{9}{32}}$

$\alpha = \dfrac{2\pi \times 3 \times 10^6}{3 \times 10^8} \times \sqrt{\dfrac{(1)(3.2)}{2}} \times \sqrt{\sqrt{1+\left(\dfrac{9}{32}\right)^2} - 1} = 0.01565$ NP/m

c) phase constant $\rightarrow \beta = \dfrac{\omega}{c}\sqrt{\dfrac{\mu_r \varepsilon_r}{2}}\sqrt{\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} + 1}$

$\beta = \dfrac{2\pi \times 3 \times 10^6}{3 \times 10^8} \times \sqrt{\dfrac{(1)(3.2)}{2}} \times \sqrt{\sqrt{1+\left(\dfrac{9}{32}\right)^2} + 1} = 0.11348$ rad/m

d) $\eta = |\eta| e^{j\theta_\eta} \rightarrow |\eta| = \dfrac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega}\right)^2}} = \dfrac{120\pi\sqrt{\frac{\mu_r=1}{\varepsilon_r=3.2}}}{\left[1+\left(\frac{9}{32}\right)^2\right]^{\frac{1}{4}}} = 206.7714$

$\theta_\eta = \dfrac{1}{2}\tan^{-1}\left(\dfrac{\sigma}{\omega\varepsilon}\right) = \dfrac{1}{2}\tan^{-1}\left(\dfrac{9}{32}\right) = 7.854°$

$\therefore$ so $\eta = |\eta|\angle\theta_\eta = 206.7714\angle 7.854°\ \Omega$

b) Consider a material for which $\mu_r = 1$, $\varepsilon_r = 2.5$ and ~~a~~ the loss tanget is 0.12 if these three values are constant with frequency in rang 0.5 MHz $\leq f \leq 100$ MHz. Calculate a) σ at (1, 75) MHz, b) λ at (1, 75) MHz, c) v_p at (1, 75) MHz

$\mu_r = 1$ ، $\varepsilon_r = \frac{2.5}{\cancel{8.1}}$ ، $\frac{\sigma}{\omega\varepsilon} = 0.12$

a) at 1 MHZ $\omega = 2\pi \times 10^6 \rightarrow \frac{\sigma}{\omega\varepsilon} = 0.12 \rightarrow \sigma = 0.12\,\omega\varepsilon$

$\sigma = 0.12\,(2\pi \times 10^6)\left(2.5 \times \frac{10^{-9}}{36\pi}\right) = 1.666 \times 10^{-5}$ S/m

at 75 MHZ $\omega = 2\pi \times 10^6 \times 75$ So $\sigma = 1.25 \times 10^{-3}$ S/m

b) at 1 MHZ $\omega = 2\pi \times 10^6$ ، $\frac{\sigma}{\omega\varepsilon} = 0.12$

$\beta = \frac{\omega}{c}\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1} = \frac{2\pi \times 10^6}{3 \times 10^8} \times \sqrt{\frac{2.5}{2}} \times \sqrt{\sqrt{1+(0.12)^2}+1} = 0.033174$ rad/m

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.033174} = 189.4$ m

at 75 MHZ $\omega = 2\pi \times 75 \times 10^6$ ، $\frac{\sigma}{\omega\varepsilon} = 0.12$

بالتعويض في نفس المعادلة السابقة مع تغير قيمة التردد نحصل على

$\beta = 2.4870$ rad/m $\longrightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.4880} = 2.5253$ m

c) at 1 MHZ $\omega = 2\pi \times 10^6$ rad/s ، $\beta = 0.033174$ rad/m

$V_p = \frac{\omega}{\beta} = 1.894 \times 10^8$ m/s

at 75 MHZ $\omega = 2\pi \times 10^6$ rad/s ، $\beta = 2.4880$ rad/m

$V_p = \frac{\omega}{\beta} = 1.894 \times 10^8$ m/s

Q2 At frequencies of 1 ، 100 ، 3000 MHZ the dielectric constant of ice made from pure water has values of 4.15 ، 3.45 ، 3.20 على التوالي while the loss tagnet is 0.12 ، 0.035 ، 0.0009 على التوالي ، If a uniform plane wave with an amplitude of 100 V/m at Z=0 is propagating through such ice, find the time average power density at $Z=0$ and $Z=10m$ for each frequency

1) $1\,MHZ$ $\omega = 2\pi \times 10^6$, $\varepsilon_r = 4.15$ → ε_r is dielectric constant

$\frac{\sigma}{\omega\varepsilon} = 0.12$

$\eta = |\eta| e^{j\theta_\eta} \rightarrow |\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{\frac{1}{4}}} = \frac{120\pi \sqrt{\frac{1}{4.15}}}{\left[1+(0.12)^2\right]^{\frac{1}{4}}} = 184.397$

in general $E = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \vec{a_x}$ V/m

$E_0 e^{-\alpha z}$: Amplitude → $\alpha = 0$ → Amplitud $= E_0 e^{0} = E_0$

$P_{avg} = \frac{(E_0 e^{-\alpha z})^2}{2|\eta|}$ $\boxed{Z=0}$ $= \frac{100^2}{2 \times 184.397} = 27.115$ W/m^2

at $Z = 10m$, $\alpha \xrightarrow{\text{من القانون}} = 2.555 \times 10^{-3}$ Np/m

So $P_{avg} = \frac{(E_0 e^{-\alpha z})^2}{2|\eta|} = \frac{100^2 \times e^{-2.555 \times 10^{-3} \times 10 \times 2}}{2 \times 184.397} = 25.767$ W/m^2

2) $100\,MHZ$ $\omega = 2\pi \times 100 \times 10^6$, $\varepsilon_r = 3.45$, $\frac{\sigma}{\omega\varepsilon} = 0.035$

$\alpha \xrightarrow{\text{قانون}} = 0.068067$ $|\eta| \xrightarrow{\text{قانون}} = 202.9030$

at $z=0$ $P_{avg} = \frac{100^2}{2 \times 202.9030} = 24.642$ W/m^2

at $Z = 10$ → $P_{avg} = \frac{100^2 e^{-0.068 \times 10 \times 2}}{2 \times 202.9030} = 6.324$ W/m^2

3) $3000\,MHZ$ $\omega = 2\pi\, 3000 \times 10^6$, $\varepsilon_r = 3.2$, $\frac{\sigma}{\omega\varepsilon} = 0.0009$ ← يمكن إهماله ≃ صفر

$\alpha \xrightarrow{\text{قانون}} = 0.05057$, $|\eta| \xrightarrow{\text{قانون}} = 210.7443$

at $Z=0$ $P_{avg} = \frac{100^2}{2 \times 210.7443} = 23.725$ W/m^2

at $Z=10$ $P_{avg} = \frac{100^2\, e^{-0.05057 \times 10 \times 2}}{2 \times 210.7443} = 8.6291$ W/m^2

Q3 a) The phasor magnetic field intensity for A 400 MHz uniform plane wave propagating in a certain lossless material is $(2a_y - j5a_z)e^{-j25x}$ A/m Known that maximum Amplitude of E is 1500 V/m : find β, η, λ, V_p, ε_r, μ_r and $H(x,y,z,t)$

1 $H = (2a_y - j5a_z)e^{-j25x}$

$= (\;a_y - j\;a_z)e^{-j\beta x} \longrightarrow \beta = 25$ rad/m

2 $\eta = \frac{E_0}{H_0} \longrightarrow E_0 = 1500$ V/m , $|H| = \sqrt{2^2+5^2} = \sqrt{29}$

$\eta = \frac{1500}{\sqrt{29}} = 278.543\ \Omega$

3 $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{25} = 0.2513\text{ m} \longrightarrow 25.13$ cm

4 $V_p = \frac{\omega}{\beta} = \frac{2\pi \times 400\times10^6}{25} = 1.005\times10^8$ m/s

5 - 6 $\varepsilon_r = ?$, $\mu_r = ?$

[1] $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} \rightarrow \sqrt{\mu_r\varepsilon_r} = \frac{\beta c}{\omega}$

So $\mu_r\varepsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = \left(\frac{25\times3\times10^8}{2\pi\times400\times10^6}\right)^2 = 8.905$

[2] $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} \rightarrow \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta}{120\pi}$

So $\frac{\mu_r}{\varepsilon_r} = \left(\frac{\eta}{120\pi}\right)^2 = \left(\frac{278.543}{120\pi}\right)^2 = 0.5459$

$\therefore\ \mu_r\varepsilon_r = 8.905$ [1] , $\frac{\mu_r}{\varepsilon_r} = 0.5459$ [2]

بضرب طرفي المعادلة [1] في $\frac{\mu_r}{\varepsilon_r}$

$\mu_r^2 = 8.905\ \frac{\mu_r}{\varepsilon_r}$

$= (8.905)(0.5459) \rightarrow \mu_r = \sqrt{4.861} = 2.204$

$\varepsilon_r = \frac{8.905}{\mu_r} = 4.04$ من [1] بالتعويض

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7 $H(x,y,z,t) = (2a_y - j\sqrt{5}a_z)e^{-j2\sqrt{5}x}e^{j\omega t} = (2a_y - j\sqrt{5}a_z)e^{j(\omega t - 2\sqrt{5}x)}$

$= (2a_y - j\sqrt{5}a_z)(\cos(\omega t - 2\sqrt{5}x) + j\sin(\omega t - 2\sqrt{5}x))$

$= 2\cos(\omega t - 2\sqrt{5}x)a_y + 2j\sin(\omega t - 2\sqrt{5}x)a_y - j\sqrt{5}\cos(\omega t - 2\sqrt{5}x)a_z$

$- j^2\sqrt{5}\sin(\omega t - 2\sqrt{5}x)a_z$ $\boxed{j^2 = -1}$

$= 2\cos(\omega t - 2\sqrt{5}x)a_y + 2j\sin(\omega t - 2\sqrt{5}x)a_y - j\sqrt{5}\cos(\omega t - 2\sqrt{5}x)a_z$

$+ \sqrt{5}\sin(\omega t - 2\sqrt{5}x)a_z$

بأخذ الجزء الحقيقي من المعادلة

$H(x,y,z,t) = 2\cos(\omega t - \beta x)\vec{a_y} + \sqrt{5}\sin(\omega t - \beta x)\vec{a_z}$ A/m

بالتعويض في القيم $= 2\cos(2\pi \times 400 \times 10^6 t - 2\sqrt{5}x)\vec{a_y} + \sqrt{5}\sin(2\pi \times 400 \times 10^6 t - 2\sqrt{5}x)\vec{a_z}$

b) Given a 20-MHZ uniform plane wave with $H_s = (6a_x - j2a_y)e^{-jz}$ A/m assume propagation in a losseless meduim Characterized by $\varepsilon_r = 5$ and unknown μ_r. find a) λ, V_p, μ_r and η

b) determine E at the origin at t = 20 ns

1 $H_s = (6a_x - j2a_y)e^{-j\beta z}$ A/m $\longrightarrow \beta = 1$ rad/m

$\varepsilon_r = 5$ $\lambda = \frac{2\pi}{\beta} = 2\pi = 6.28$ m

2 $\omega = 2\pi \times 20 \times 10^6$ $V_p = \frac{\omega}{\beta} = \omega = 2\pi \times 20 \times 10^6 = 40\pi \times 10^6$ m/s

3 $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{5}\sqrt{\mu_r}$

$\mu_r = \left(\frac{\beta c}{\omega\sqrt{5}}\right)^2 = \frac{3 \times 10^8}{2\pi \times 20 \times 10^6 \times \sqrt{5}} = 1.1398$

4 $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1.1398}{5}} = 179.995\ \Omega$

5 $H_s = (6a_x - 2ja_y)e^{-jz} \Rightarrow H(z,t) = Re[H_s e^{j\omega t}] = (6a_x - j2a_y)e^{-jz}e^{j\omega t}$

$= 6\cos(\omega t - z)\vec{a_x} - 2\sin(\omega t - z)\vec{a_y}$ $E = \eta H$

$E(z,t) = -\eta 6\cos(\omega t - z)\vec{a_y} + \eta 2\sin(\omega t - z)\vec{a_x} = -1.079\cos(\omega t - z)\vec{a_y} + 0.359\sin(\omega t - z)\vec{a_x}$ KV/m

$E(0, 20ns) = 0.8729\vec{a_y} + 0.211\vec{a_x}$ KV/m

Q4 Let $\eta = 250 + j30\ \Omega$ and $jk = 0.2 + j2\ m^{-1}$ for a uniform plane wave propagating in the $\vec{a_z}$ direction in a dielectric have some finite conductivity if $|E_s| = 400$ V/m at $z=0$ Find a) $P_{z,avg}$ at $z=0$ & $z=60$ cm & b) the average ohmic power dissipation in watt per cubic meter at $z=60$ cm

a) $\eta = 250 + j30\ \Omega \rightarrow 251.79 \angle -0.119 \rightarrow |\eta| = 251.79$

$jk = \gamma = 0.2 + j2\ m^{-1} = \alpha + j\beta \rightarrow \alpha = 0.2$ & $\beta = 2$

$E_s = |E_s| e^{-jkz} = |E_s| e^{-j\gamma z} = \underbrace{|E_s| e^{-\alpha z}}_{\text{Amplitude}} e^{-j\beta z}$

$$P_{z,avg} = \frac{(|E_s| e^{-\alpha z})^2}{2|\eta|} \vec{a_z}$$

at $z=0$: $P_{z,avg} = \dfrac{(400\, e^{-0.2(0)})^2}{2 \times 251.79} = \dfrac{400^2}{503.58} = 317.72\ \vec{a_z}$ W/m²

at $z = 60$ cm $P_{z,avg} = \dfrac{400 \cdot e^{-0.2 \times 60 \times 10^{-2} \times 2}}{2 \times 251.79} = 249.93\ \vec{a_z}$ W/m²

b) $-\nabla \cdot P_{z\,av} = \langle J \cdot E \rangle$

$\langle J \cdot E \rangle = -\nabla \cdot P_{z\,avg} = -\frac{d}{dz} 315\, e^{-2(0.2)z} = (0.4)(315) e^{-2(0.2)z} = 126\, e^{-0.4z}$ W/m³

at $z = 60$ cm $\langle J \cdot E \rangle = 99.1$ W/m³ $\rightarrow \langle J \cdot E \rangle = \sigma \langle E^2 \rangle$

$\sigma = \omega \varepsilon'' \rightarrow \sigma = Re\left[\frac{jk}{\eta}\right] = Re\left[\frac{0.2 + j2}{250 + j30}\right] = 1.74 \times 10^{-3}$ S/m

dissipated power $\sigma \langle E^2 \rangle = 1.74 \times 10^{-3} \left(\frac{1}{2}\right)(400\, e^{-0.2z})^2$

$Re\left[\frac{1}{\eta^*}\right] = \frac{\sigma}{2\alpha}$ at $z = 60$ cm

evaluates 109 W/m³

Q_5 In a transmition line fillid with a lossless dielectric ($\varepsilon_r = 4.5$, $\mu_r = 1$)

$$E = \frac{40}{\rho} \sin(\omega t - 2z)\, a_\rho \ \ V/m$$

Find a) ω and H, b) the poying vector, c) the total time average power crossing the surface $z = 1m$, $2mm < \rho < 3mm$, $0 < \phi < 2\pi$

$\varepsilon_r = 4.5$, $\mu_r = 1$, $\beta = 2$

a) $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r} \rightarrow \omega = \frac{\beta c}{\sqrt{\varepsilon_r}} = 2.828 \times 10^8\ rad/s$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{1}{4.5}} = 177.715\ \Omega$ $\quad H = \frac{E}{\eta}$

$H = \frac{40}{\eta\rho} \sin(\omega t - 2z)\, a_\phi\ A/m = \frac{0.225}{\rho} \sin(\omega t - 2z)\, a_\phi\ A/m$

b) $\vec{P} = \frac{E_0^2}{\eta} \sin^2(\omega t - \beta z)\, \vec{a_z} = \frac{9.003}{\rho^2} \sin^2(\omega t - 2z)\, \vec{a_z}\ W/m^2$

c) $P_{avg} = \frac{E_0^2}{2\eta} \vec{a_z} = \frac{4.5}{\rho^2} \vec{a_z}$

$P_{avg} = \oint_s \vec{P_{avg}} \cdot d\vec{s} = \frac{4.5\vec{a_z}}{\rho^2} \int_2^3 \rho\, d\rho \cdot \int_0^{2\pi} d\phi\ \vec{a_z}$

$= 4.5\vec{a_z} \int_2^3 \frac{1}{\rho} d\rho \cdot \int_0^{2\pi} d\phi = 4.5\vec{a_z} [\ln\rho]_2^3 \cdot [\phi]_0^{2\pi}$

$= [4.5\vec{a_z}] \left[[\ln 3 - \ln 2][2\pi]\, a_z\right] = 11.4642\ W$

Q_6 a) The plane wave $E = 30\cos(\omega t - z)\, a_x$ V/m in air normally hits a lossless medium ($\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$) at $z = 0$ a) find Γ, T, S

b) Calculate the reflected electric and magnetic fields

1- a) $B_1 = 1$ $\quad \omega = \beta c = 3\times10^8$ rad/m $\quad \eta_1 = 120\pi$

$B_2 = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = 1\sqrt{4} = 2$, $\eta_2 = 120\pi\sqrt{\frac{1}{4}} = 60\pi$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-1}{3} \longrightarrow \tau = 1 - \frac{1}{3} = \frac{2}{3}$

$S = SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2$

b) $E_i = 30\cos(wt - z)\vec{a_x}$ V/m

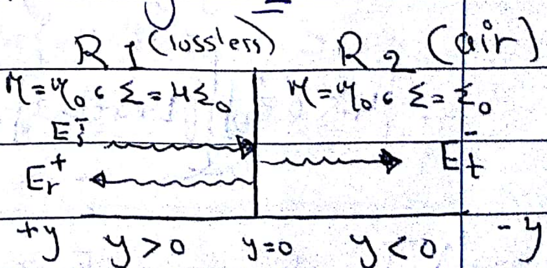
* $E_r = \Gamma E_i = -10\cos(wt + z)\vec{a_x}$ V/m

$H_i = \frac{30}{\eta_1}\cos(wt - z)\vec{a_y}$ A/m $= 79.57\cos(wt - z)\vec{a_y}$ mA/m

* $H_r = +\frac{10}{\eta_1}\cos(wt + z)\vec{a_y} = +26.52\cos(wt - z)\vec{a_y}$ mA/m

Q6 b) Region 1 is lossess medium for which $y \geq 0$ $\mu = \mu_0$ $\varepsilon = 4\varepsilon_0$ whereas in region 2 is free space $y \leq 0$ if a plane wave $E = 5\cos(10^8 t + By)\vec{a_z}$ V/m exists in region 1 find a) the total electric field component of the wave in region 2 b) the time average poynting vector in region 1 c) the time average poynting vector in region 2

$E_i = 5\cos(10^8 t + By)\vec{a_z}$ V/m



$w_1 = w_2$ $\quad w = 10^8 \rightarrow B = \frac{w}{c}\sqrt{\mu_r\varepsilon_r} = \frac{10^8}{3\times10^8}\sqrt{4} = \frac{2}{3}$ rad/m

$\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{1}{4}} = 60\pi$

$B_2 = \frac{w}{c} = \frac{1}{3}$, $\eta_2 = \eta_0 = 120\pi$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1}{3} \longrightarrow \tau = 1 + \Gamma = \frac{4}{3}$

a) in Region 2 $E_{total} = E_{transmit} = \tau E_i = \frac{20}{3}\cos(10^8 t + \frac{1}{3}y)\vec{a_z}$ V/m

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b) in Region 1: $E_1 = E_{total} = E_i + E_r$

$E_r = \Gamma E_i = \frac{5}{3}\cos(10^8 t - \frac{2}{3}y)\, \vec{a_z}$ V/m

$E_1 = 5\cos(10^8 t + \frac{2}{3}y)\, \vec{a_z} + \frac{5}{3}\cos(10^8 t - \frac{2}{3}y)\, \vec{a_z}$ V/m

$P_{avg} = \frac{E_{io}^2}{2\eta_1}(-a_y) + \frac{E_{ro}^2}{2\eta_1} a_y = -\frac{(5)^2}{120\pi}\vec{a_y} + \frac{(\frac{5}{3})^2}{120\pi} a_y = -0.05894 a_y$ W/m^2

c) in Region 2: $E_2 = E_{total} = E_{transmit} = \frac{20}{3}\cos(10^8 t + \frac{1}{3}y)\, \vec{a_z}$ V/m

$P_{avg} = \frac{E_t^2}{2\eta_2}(-a_y) = \frac{(\frac{20}{3})^2}{240\pi}(-a_y) = -0.05894 a_y$ W/m^2

Q7 a polarized wave is incident from air to polystyrene with $\mu = \mu_0$, $\varepsilon = 2.6\varepsilon_0$ at Brewster angle. Determine the transmision Angle

$\theta_B \rightarrow$ is Brewster angle

$\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ parallel , $\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$ perpendicular (ليس)

$[\varepsilon_1 = \varepsilon_0,\ \mu_1 = \mu_0,\ \eta_1 = 120\pi]$, $[\varepsilon_2 = 2.6\varepsilon_0,\ \mu_2 = \mu_0,\ \eta_2 = \eta_1\sqrt{\frac{\mu_2}{\varepsilon_{r2}}}]$

So: $\tan\theta_{B\parallel} = \sqrt{\frac{2.6\varepsilon_0}{\varepsilon_0}} = 58.193^\circ$

$\theta_t \rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel} \rightarrow \cos\theta_t = \frac{\eta_1 \cos\theta_{B\parallel}}{\eta_2} = \frac{\eta_1 \cos\theta_{B\parallel}}{\eta_1\sqrt{\frac{1}{2.6}}}$

$\cos\theta_t = 0.8498 \rightarrow \theta_t = 31.81^\circ$

Q8 which of the following media may be treated as conducting at 8MHz

a) wet marshy soil ($\varepsilon = 15\varepsilon_0$, $\mu = \mu_0$, $\sigma = 10^{-2}$ S/m)

b) Instrinsic germanium ($\varepsilon = 16\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0.025$ S/m)

c) Sea water ($\varepsilon = 81\varepsilon_0$, $\mu = \mu_0$, $\sigma = 25$ S/m)

Loss tangent $\frac{\sigma}{\omega\varepsilon}$ ← حساب (يمكن معرفة طبيعة الوسط او المادة من خلال)

حيث → $\frac{\sigma}{\omega\varepsilon} \ll 1$ $\xrightarrow{\text{يعني}}$ lossless (عوازل تامة)

→ $\frac{\sigma}{\omega\varepsilon} \simeq 1$ $\xrightarrow{\text{تقريبي}}$ lossy

→ $\frac{\sigma}{\omega\varepsilon} \gg 1$ $\xrightarrow{\text{يعني}}$ Conducting (good Conductor)

$\therefore$ $\omega = 2\pi \times 8\times10^{6}$ — قريب من $\frac{1}{2}$

a) $\varepsilon = 15\varepsilon_0$, $\mu = \mu_0$, $\sigma = 10^{-2}$ $\rightarrow$ $\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{2\pi\times8\times10^{6}\times15\times\frac{10^{-9}}{36\pi}} = \frac{3}{2} = 1.5$ Lossy

قريب من $\frac{1}{2}$

b) $\varepsilon = 16\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0.025$ $\rightarrow$ $\frac{\sigma}{\omega\varepsilon} = \frac{0.025}{2\pi\times8\times10^{6}\times16\times\frac{10^{-9}}{36\pi}} = 3.515$ Lossy

c) $\varepsilon = 81\varepsilon_0$, $\mu = \mu_0$, $\sigma = 25$ $\rightarrow$ $\frac{\sigma}{\omega\varepsilon} = \frac{25}{2\pi\times8\times10^{6}\times81\times\frac{10^{-9}}{36\pi}} = 694.44 \gg 1$ $\therefore$ is Conducting

Q9 An air line has Characteristic impedanc of $70\,\Omega$ and phase Constant of 3 rad/m at 100 MHZ Claculat the inductance per meter and the Capacitance per meter of the line

An air بنقدر اعتبار → كمادة عازلة → $\sigma = 0$

$R = 0 = G$ $\alpha = 0$

$\therefore$ $Z_0 = R_0 = \sqrt{\frac{L}{C}}$ [1] , $\beta = \omega\sqrt{LC}$ [2]

$\frac{R_0}{\beta} = \frac{1}{\omega C}$ بقسمة المعادلتين نحصل على

$\therefore$ $C = \frac{\beta}{\omega R_0} = \frac{3}{2\pi\times100\times10^{6}\times70} = 68.20\ \text{PF/m}$

$L = R_0^2 C \rightarrow L = (70)^2(68.20\times10^{-12}) = 334.18\ \text{nH/m}$ [I] وعن المعادلة

Q10 A uniform plane wave in air with $E = 8\cos(\omega t - 4x - 3z)\,a_y$ V/m is incident on a dielectric slab $(z \geq 0)$ with $(\mu_r = 1$, $\varepsilon_r = 2.5$, $\sigma = 0)$

Find a) The polarization of the wave

b) The angle of incident

c) The reflected E Field

d) The transmitted H Field

$E = 8\cos(\omega t - 4x - 3z)\,\vec{a_y}$ V/m

a) $= 8\, e^{-j(4x+3z-\omega t)}\,\vec{a_y}$

$= E_0\, e^{-j(\vec{K}\cdot\vec{r} - \omega t)}\,\vec{a_y}$ x-z plane , y constant

from $(4x+3z)\,\vec{a_y}$ ⟶ perpendicular polarization

b) from (a) $\vec{K}\cdot\vec{r} = 4x+3z \rightarrow \vec{K_i} = 4a_x + 3a_z$

$B_1 = |K_i| = \sqrt{4^2+3^2} = 5$

in air $\omega = |K_i|\,C = BC = 15\times10^8$ rad/m

in general $\vec{K_i} = K_{ix}\vec{a_x} + K_{iy}\vec{a_y} + K_{iz}\vec{a_z} = K_{ix}\vec{a_x} + K_{iz}\vec{a_z}$

← معلوم

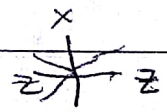
$\tan\theta_i = \frac{K_{ix}}{K_{iz}} = \frac{4}{3} = 53.13^\circ$

Or من القانون ⟶ $\cos\theta_i = a_k\cdot a_n$ حيث $\vec{K} = |K|\,\vec{a_K}$

$a_K = \frac{\vec{K}}{|K|} = \frac{4a_x+3a_z}{5} = 0.8\vec{a_x} + 0.6\vec{a_z}$

هو اتجاه المحور الافقي — تساوي ⟶

an ⟶ $\vec{a_z}$



$\therefore \cos\theta_i = a_K\cdot a_n = (0.8\vec{a_x} + 0.6\vec{a_z})\cdot\vec{a_z} = 0.6$

$\theta_i = \cos^{-1} 0.6 = 53.13^\circ$

c) $K_i = (K_{ix}\vec{a_x} + K_{iz}\vec{a_z}) \rightarrow K_r = K_{rx}\vec{a_x} - K_{rz}\vec{a_z}$

$5 = K_i = K_r$, $\theta_i = \theta_r = 53.13^\circ$

$\vec{K_r} = 4\vec{a_x} - 3\vec{a_z}$ $\quad \vec{K_r}\cdot\vec{r} = 4x - 3z$

$n_1\sin\theta_i = n_2\sin\theta_t \rightarrow (n_1 = 1$ air$)$, $n_2 = C\sqrt{\mu_2\varepsilon_2} = 1.5811$

$\theta_t = \sin^{-1}\left(\frac{\sin\theta_i}{n_2}\right) = 30.39^\circ$

$\Gamma_\perp = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} = -0.389$; $T_\perp = 1 - 0.389 = 0.611$

$E_r = \Gamma E_i = -3.112 \cos(\omega t - 4x + 3z)\, \vec{a_y}$ V/m

d) $K_{tx} = K_t \sin\Theta_t$, $K_{tz} = K_t \cos\Theta_t$

$\Theta_t = 30.39^\circ$, $K_t = \beta_2 = \frac{\omega}{c}\sqrt{\mu_{r2}\varepsilon_{r2}} = 7.905$

$K_{tx} = 7.905 \sin 30.39^\circ = 4$, $K_{tz} = 7.905 \cos 30.39^\circ = 6.818$

$\vec{K_t} = 4\vec{a_x} + 6.818\vec{a_z}$

$E_t = \Gamma' E_i = 4.88 \cos(\omega t - 4x - 6.818z)\, \vec{a_y}$ V/m

$H_t = \frac{1}{\mu_2 \omega} \vec{K_t} \times E_t = \frac{4\vec{a_x} + 6.818\vec{a_z}}{4\pi\times10^{-7}\times15\times10^{8}} \times 4.888 \cos(\omega t - \vec{K}\cdot\vec{r})\, \vec{a_y}$

$= (-17.65 a_x + 10.374 a_z) \cos(15\times10^{8} t - 4x - 6.818z)$ mA/m

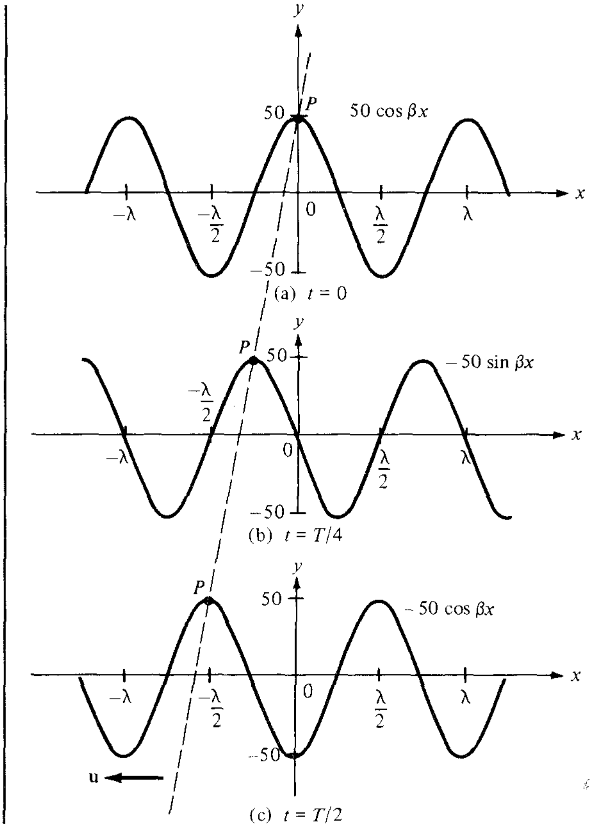


Figure 10.3 For Example 10.1; wave travels along $-\mathbf{a}_x$.

PRACTICE EXERCISE 10.1

In free space, $\mathbf{H} = 0.1 \cos (2 \times 10^8 t - kx)\, \mathbf{a}_y$ A/m. Calculate

(a) k, λ, and T

(b) The time t_1 it takes the wave to travel $\lambda/8$

(c) Sketch the wave at time t_1.

Answer: (a) 0.667 rad/m, 9.425 m, 31.42 ns, (b) 3.927 ns, (c) see Figure 10.4.

10.3 WAVE PROPAGATION IN LOSSY DIELECTRICS

As mentioned in Section 10.1, wave propagation in lossy dielectrics is a general case from which wave propagation in other types of media can be derived as special cases. Therefore, this section is foundational to the next three sections.

We define the *surface or skin resistance* R_s (in Ω/m^2) as the real part of the η for a good conductor. Thus from eq. (10.55)

$$\boxed{R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}} \tag{10.56}$$

This is the resistance of a unit width and unit length of the conductor. It is equivalent to the dc resistance for a unit length of the conductor having cross-sectional area $1 \times \delta$. Thus for a given width w and length ℓ, the ac resistance is calculated using the familiar dc resistance relation of eq. (5.16) and assuming a uniform current flow in the conductor of thickness δ, that is,

$$\boxed{R_{\text{ac}} = \frac{\ell}{\sigma\delta w} = \frac{R_s \ell}{w}} \tag{10.57}$$

where $S \simeq \delta w$. For a conductor wire of radius a (see Figure 10.9), $w = 2\pi a$, so

$$\frac{R_{\text{ac}}}{R_{\text{dc}}} = \frac{\dfrac{\ell}{\sigma 2\pi a \delta}}{\dfrac{\ell}{\sigma \pi a^2}} = \frac{a}{2\delta}$$

Since $\delta \ll a$ at high frequencies, this shows that R_{ac} is far greater than R_{dc}. In general, the ratio of the ac to the dc resistance starts at 1.0 for dc and very low frequencies and increases as the frequency increases. Also, although the bulk of the current is nonuniformly distributed over a thickness of 5δ of the conductor, the power loss is the same as though it were uniformly distributed over a thickness of δ and zero elsewhere. This is one more reason why δ is referred to as the skin depth.

EXAMPLE 10.2

A lossy dielectric has an intrinsic impedance of $200 \angle 30^\circ\ \Omega$ at a particular frequency. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component

$$\mathbf{H} = 10\, e^{-\alpha x} \cos\left(\omega t - \frac{1}{2}x\right) \mathbf{a}_y \text{ A/m}$$

find $\mathbf{E}$ and α. Determine the skin depth and wave polarization.

Solution:

The given wave travels along $\mathbf{a}_x$ so that $\mathbf{a}_k = \mathbf{a}_x$; $\mathbf{a}_H = \mathbf{a}_y$, so

$$-\mathbf{a}_E = \mathbf{a}_k \times \mathbf{a}_H = \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

or

$$\mathbf{a}_E = -\mathbf{a}_z$$

Also $H_o = 10$, so

$$\frac{E_o}{H_o} = \eta = 200\,\angle 30^\circ = 200\, e^{j\pi/6} \rightarrow E_o = 2000 e^{j\pi/6}$$

Except for the amplitude and phase difference, **E** and **H** always have the same form. Hence

$$\mathbf{E} = \text{Re}\,(2000 e^{j\pi/6} e^{-\gamma x} e^{j\omega t} \mathbf{a}_E)$$

or

$$\mathbf{E} = -2e^{-\alpha x} \cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right) \mathbf{a}_z \text{ kV/m}$$

Knowing that $\beta = 1/2$, we need to determine α. Since

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

and

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1}{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1}\right]^{1/2}$$

But $\dfrac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60^\circ = \sqrt{3}$. Hence,

$$\frac{\alpha}{\beta} = \left[\frac{2 - 1}{2 + 1}\right]^{1/2} = \frac{1}{\sqrt{3}}$$

or

$$\alpha = \frac{\beta}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2887 \text{ Np/m}$$

and

$$\delta = \frac{1}{\alpha} = 2\sqrt{3} = 3.464 \text{ m}$$

The wave has an E_z component; hence it is polarized along the z-direction.

PRACTICE EXERCISE 10.2

A plane wave propagating through a medium with $\varepsilon_r = 8$, $\mu_r = 2$ has $\mathbf{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \mathbf{a}_x$ V/m. Determine

(a) β

(b) The loss tangent

(c) Wave impedance

(d) Wave velocity

(e) **H** field

Answer: (a) 1.374 rad/m, (b) 0.5154, (c) $177.72 \angle 13.63^\circ\ \Omega$, (d) 7.278×10^7 m/s, (e) $2.817 e^{-z/3} \sin(10^8 t - \beta z - 13.63^\circ)\mathbf{a}_y$ mA/m.

EXAMPLE 10.3

In a lossless medium for which $\eta = 60\pi$, $\mu_r = 1$, and $\mathbf{H} = -0.1 \cos(\omega t - z)\, \mathbf{a}_x + 0.5 \sin(\omega t - z)\mathbf{a}_y$ A/m, calculate ε_r, ω, and $\mathbf{E}$.

Solution:

In this case, $\sigma = 0$, $\alpha = 0$, and $\beta = 1$, so

$$\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

or

$$\sqrt{\varepsilon_r} = \frac{120\pi}{\eta} = \frac{120\pi}{60\pi} = 2 \quad \rightarrow \quad \varepsilon_r = 4$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\, \sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{4} = \frac{2\omega}{c}$$

or

$$\omega = \frac{\beta c}{2} = \frac{1\,(3 \times 10^8)}{2} = 1.5 \times 10^8 \text{ rad/s}$$

From the given **H** field, **E** can be calculated in two ways: using the techniques (based on Maxwell's equations) developed in this chapter or directly using Maxwell's equations as in the last chapter.

Method 1: To use the techniques developed in this chapter, we let

$$\mathbf{E} = \mathbf{H}_1 + \mathbf{H}_2$$

where $\mathbf{H}_1 = -0.1 \cos(\omega t - z)\, \mathbf{a}_x$ and $\mathbf{H}_2 = 0.5 \sin(\omega t - z)\, \mathbf{a}_y$ and the corresponding electric field

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

where $\mathbf{E}_1 = E_{1o} \cos(\omega t - z)\, \mathbf{a}_{E_1}$ and $\mathbf{E}_2 = E_{2o} \sin(\omega t - z)\, \mathbf{a}_{E_2}$. Notice that although $\mathbf{H}$ has components along $\mathbf{a}_x$ and $\mathbf{a}_y$, it has no component along the direction of propagation; it is therefore a TEM wave.

For $\mathbf{E}_1$,

$$\mathbf{a}_{E_1} = -(\mathbf{a}_k \times \mathbf{a}_{H_1}) = -(\mathbf{a}_z \times -\mathbf{a}_x) = \mathbf{a}_y$$

$$E_{1o} = \eta H_{1o} = 60\pi\,(0.1) = 6\pi$$

Hence

$$\mathbf{E}_1 = 6\pi \cos(\omega t - z)\, \mathbf{a}_y$$

For $\mathbf{E}_2$,

$$\mathbf{a}_{E_2} = -(\mathbf{a}_k \times \mathbf{a}_{H_2}) = -(\mathbf{a}_z \times \mathbf{a}_y) = \mathbf{a}_x$$

$$E_{2o} = \eta H_{2o} = 60\pi\,(0.5) = 30\pi$$

Hence

$$\mathbf{E}_2 = 30\pi \sin(\omega t - z)\mathbf{a}_x$$

Adding $\mathbf{E}_1$ and $\mathbf{E}_2$ gives $\mathbf{E}$; that is,

$$\mathbf{E} = 94.25 \sin(1.5 \times 10^8 t - z)\, \mathbf{a}_x + 18.85 \cos(1.5 \times 10^8 t - z)\, \mathbf{a}_y \text{ V/m}$$

Method 2: We may apply Maxwell's equations directly.

$$\nabla \times \mathbf{H} = \underbrace{\sigma \mathbf{E}}_{0} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad \rightarrow \quad \mathbf{E} = \frac{1}{\varepsilon} \int \nabla \times H\, dt$$

because $\sigma = 0$. But

$$\nabla \times \mathbf{H} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x(z) & H_y(z) & 0 \end{vmatrix} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x + \frac{\partial H_x}{\partial z}\mathbf{a}_y$$
$$= H_{2o} \cos(\omega t - z)\, \mathbf{a}_x + H_{1o} \sin(\omega t - z)\mathbf{a}_y$$

where $H_{1o} = -0.1$ and $H_{2o} = 0.5$. Hence

$$\mathbf{E} = \frac{1}{\varepsilon} \int \nabla \times \mathbf{H}\, dt = \frac{H_{2o}}{\varepsilon\omega} \sin(\omega t - z)\, \mathbf{a}_x - \frac{H_{1o}}{\varepsilon\omega} \cos(\omega t - z)\, \mathbf{a}_y$$
$$= 94.25 \sin(\omega t - z)\, \mathbf{a}_x + 18.85 \cos(\omega t - z)\, \mathbf{a}_y \text{ V/m}$$

as expected.

PRACTICE EXERCISE 10.3

A plane wave in a nonmagnetic medium has $\mathbf{E} = 50 \sin(10^8 t + 2z)\, \mathbf{a}_y$ V/m. Find

(a) The direction of wave propagation

(b) $\lambda, f,$ and ε_r

(c) $\mathbf{H}$

Answer: (a) along $-z$ direction, (b) 3.142 m, 15.92 MHz, 36, (c) 0.7958 $\sin(10^8 t + 2z)\, \mathbf{a}_x$ A/m.

EXAMPLE 10.4

A uniform plane wave propagating in a medium has

$$\mathbf{E} = 2e^{-\alpha z} \sin(10^8 t - \beta z)\, \mathbf{a}_y \text{ V/m.}$$

If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$, and $\sigma = 3$ mhos/m, find α, β, and $\mathbf{H}$.

Solution:

We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor.

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^8 \times 1 \times \dfrac{10^{-9}}{36\pi}} = 3393 \gg 1$$

showing that the medium may be regarded as a good conductor at the frequency of operation. Hence,

$$\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \left[\frac{4\pi \times 10^{-7} \times 20(10^8)(3)}{2}\right]^{1/2}$$
$$= 61.4$$
$$\alpha = 61.4 \text{ Np/m}, \qquad \beta = 61.4 \text{ rad/m}$$

Also

$$|\eta| = \sqrt{\frac{\mu\omega}{\sigma}} = \left[\frac{4\pi \times 10^{-7} \times 20(10^8)}{3}\right]^{1/2}$$
$$= \sqrt{\frac{800\pi}{3}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 3393 \quad \rightarrow \quad \theta_\eta = 45° = \pi/4$$

Hence

$$\mathbf{H} = H_o e^{-\alpha z} \sin\left(\omega t - \beta z - \frac{\pi}{4}\right) \mathbf{a}_H$$

where

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

and

$$H_o = \frac{E_o}{|\eta|} = 2\sqrt{\frac{3}{800\pi}} = 69.1 \times 10^{-3}$$

Thus

$$\mathbf{H} = -69.1\, e^{-61.4z} \sin\left(10^8 t - 61.42z - \frac{\pi}{4}\right) \mathbf{a}_x \text{ mA/m}$$

PRACTICE EXERCISE 10.4

A plane wave traveling in the $+y$-direction in a lossy medium ($\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 10^{-2}$ mhos/m) has $\mathbf{E} = 30 \cos(10^9\pi\, t + \pi/4)\, \mathbf{a}_z$ V/m at $y = 0$. Find

(a) $\mathbf{E}$ at $y = 1$ m, $t = 2$ ns
(b) The distance traveled by the wave to have a phase shift of 10°
(c) The distance traveled by the wave to have its amplitude reduced by 40%
(d) $\mathbf{H}$ at $y = 2$ m, $t = 2$ ns

Answer: (a) $2.787\mathbf{a}_z$ V/m, (b) 8.325 mm, (c) 542 mm, (d) $-4.71\mathbf{a}_x$ mA/m

XAMPLE 10.5

A plane wave $\mathbf{E} = E_o \cos(\omega t - \beta z)\, \mathbf{a}_x$ is incident on a good conductor at $z = 0$. Find the current density in the conductor.

Solution:

Since the current density $\mathbf{J} = \sigma\mathbf{E}$, we expect $\mathbf{J}$ to satisfy the wave equation in eq. (10.17), that is,

$$\nabla^2 \mathbf{J}_s - \gamma^2 \mathbf{J}_s = 0$$

Also the incident $\mathbf{E}$ has only an x-component and varies with z. Hence $\mathbf{J} = J_x(z, t)\, \mathbf{a}_x$ and

$$\frac{d^2}{dz^2} J_{sx} - \gamma^2 J_{sx} = 0$$

which is an ordinary differential equation with solution (see Case 2 of Example 6.5)

$$J_{sx} = Ae^{-\gamma z} + Be^{+\gamma z}$$

The constant B must be zero because J_{sx} is finite as $z \to \infty$. But in a good conductor, $\sigma \gg \omega\varepsilon$ so that $\alpha = \beta = 1/\delta$. Hence

$$\gamma = \alpha + j\beta = \alpha(1 + j) = \frac{(1 + j)}{\delta}$$

and

$$J_{sx} = Ae^{-z(1+j)/\delta}$$

or

$$J_{sx} = J_{sx}(0)\, e^{-z(1+j)/\delta}$$

where J_{sx} (0) is the current density on the conductor surface.

PRACTICE EXERCISE 10.5

Due to the current density of Example 10.5, find the magnitude of the total current through a strip of the conductor of infinite depth along z and width w along y.

Answer: $\dfrac{J_{sx}(0)w\delta}{\sqrt{2}}$

EXAMPLE 10.6

For the copper coaxial cable of Figure 7.12, let $a = 2$ mm, $b = 6$ mm, and $t = 1$ mm. Calculate the resistance of 2 m length of the cable at dc and at 100 MHz.

Solution:

Let

$$R = R_o + R_i$$

where R_o and R_i are the resistances of the inner and outer conductors.
At dc,

$$R_i = \frac{\ell}{\sigma S} = \frac{\ell}{\sigma \pi a^2} = \frac{2}{5.8 \times 10^7 \pi[2 \times 10^{-3}]^2} = 2.744 \text{ m}\Omega$$

$$\begin{aligned} R_o &= \frac{\ell}{\sigma S} = \frac{\ell}{\sigma\pi[[b + t]^2 - b^2]} = \frac{\ell}{\sigma\pi[t^2 + 2bt]} \\ &= \frac{2}{5.8 \times 10^7 \pi\, [1 + 12] \times 10^{-6}} \\ &= 0.8429 \text{ m}\Omega \end{aligned}$$

Hence $R_{dc} = 2.744 + 0.8429 = 3.587$ mΩ

At $f = 100$ MHz,

$$\begin{aligned} R_i &= \frac{R_s \ell}{w} = \frac{\ell}{\sigma\delta 2\pi a} = \frac{\ell}{2\pi a}\sqrt{\frac{\pi f \mu}{\sigma}} \\ &= \frac{2}{2\pi \times 2 \times 10^{-3}}\sqrt{\frac{\pi \times 10^8 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} \\ &= 0.41\ \Omega \end{aligned}$$

Since $\delta = 6.6\ \mu\text{m} \ll t = 1$ mm, $w = 2\pi b$ for the outer conductor. Hence,

$$\begin{aligned} R_o &= \frac{R_s \ell}{w} = \frac{\ell}{2\pi b}\sqrt{\frac{\pi f \mu}{\sigma}} \\ &= \frac{2}{2\pi \times 6 \times 10^{-3}}\sqrt{\frac{\pi \times 10^8 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} \\ &= 0.1384\ \Omega \end{aligned}$$

Hence,

$$R_{ac} = 0.41 + 0.1384 = 0.5484\ \Omega$$

which is about 150 times greater than R_{dc}. Thus, for the same effective current i, the ohmic loss (i^2R) of the cable at 100 MHz is far greater than the dc power loss by a factor of 150.

PRACTICE EXERCISE 10.6

For an aluminum wire having a diameter 2.6 mm, calculate the ratio of ac to dc resistance at

(a) 10 MHz

(b) 2 GHz

Answer: (a) 24.16, (b) 341.7.

10.7 POWER AND THE POYNTING VECTOR

As mentioned before, energy can be transported from one point (where a transmitter is located) to another point (with a receiver) by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{10.58a}$$

$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{10.58b}$$

and

$$\begin{aligned}\mathcal{P}(z, t) &= \frac{E_o^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\, \mathbf{a}_z \\ &= \frac{E_o^2}{2|\eta|} e^{-2\alpha z} [\cos \theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)]\, \mathbf{a}_z\end{aligned} \tag{10.66}$$

since $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$. To determine the time-average Poynting vector $\mathcal{P}_{ave}(z)$ (in W/m^2), which is of more practical value than the instantaneous Poynting vector $\mathcal{P}(z, t)$, we integrate eq. (10.66) over the period $T = 2\pi/\omega$; that is,

$$\mathcal{P}_{ave}(z) = \frac{1}{T} \int_0^T \mathcal{P}(z, t)\, dt \tag{10.67}$$

It can be shown (see Prob. 10.28) that this is equivalent to

$$\boxed{\mathcal{P}_{ave}(z) = \frac{1}{2} \mathrm{Re}\,(\mathbf{E}_s \times \mathbf{H}_s^*)} \tag{10.68}$$

By substituting eq. (10.66) into eq. (10.67), we obtain

$$\boxed{\mathcal{P}_{ave}(z) = \frac{E_o^2}{2|\eta|} e^{-2\alpha z} \cos \theta_\eta\, \mathbf{a}_z} \tag{10.69}$$

The total time-average power crossing a given surface S is given by

$$\boxed{P_{ave} = \int_S \mathcal{P}_{ave} \cdot d\mathbf{S}} \tag{10.70}$$

We should note the difference between $\mathcal{P}$, $\mathcal{P}_{ave}$, and P_{ave}. $\mathcal{P}(x, y, z, t)$ is the Poynting vector in watts/meter and is time varying. $\mathcal{P}_{ave}(x, y, z)$ also in watts/meter is the time average of the Poynting vector $\mathcal{P}$; it is a vector but is time invariant. P_{ave} is a total time-average power through a surface in watts; it is a scalar.

EXAMPLE 10.7

In a nonmagnetic medium

$$\mathbf{E} = 4 \sin(2\pi \times 10^7 t - 0.8x)\, \mathbf{a}_z \text{ V/m}$$

Find

(a) ε_r, η
(b) The time-average power carried by the wave
(c) The total power crossing 100 cm^2 of plane $2x + y = 5$

Solution:

(a) Since $\alpha = 0$ and $\beta \neq \omega/c$, the medium is not free space but a lossless medium.

$$\beta = 0.8, \quad \omega = 2\pi \times 10^7, \quad \mu = \mu_o \text{ (nonmagnetic)}, \quad \varepsilon = \varepsilon_o\varepsilon_r$$

Hence

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

or

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{0.8(3 \times 10^8)}{2\pi \times 10^7} = \frac{12}{\pi}$$

$$\varepsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = 120\pi \cdot \frac{\pi}{12} = 10\pi^2$$
$$= 98.7\ \Omega$$

(b) $\mathcal{P} = \mathbf{E} \times \mathbf{H} = \dfrac{E_o^2}{\eta} \sin^2(\omega t - \beta x)\, \mathbf{a}_x$

$$\mathcal{P}_{ave} = \frac{1}{T}\int_0^T \mathcal{P}\, dt = \frac{E_o^2}{2\eta}\mathbf{a}_x = \frac{16}{2 \times 10\pi^2}\mathbf{a}_x$$
$$= 81\ \mathbf{a}_x \text{ mW/m}^2$$

(c) On plane $2x + y = 5$ (see Example 3.5 or 8.5),

$$\mathbf{a}_n = \frac{2\mathbf{a}_x + \mathbf{a}_y}{\sqrt{5}}$$

Hence the total power is

$$P_{ave} = \int \mathcal{P}_{ave} \cdot d\mathbf{S} = \mathcal{P}_{ave} \cdot S\, \mathbf{a}_n$$
$$= (81 \times 10^{-3}\mathbf{a}_x) \cdot (100 \times 10^{-4}) \left[\frac{2\mathbf{a}_x + \mathbf{a}_y}{\sqrt{5}}\right]$$
$$= \frac{162 \times 10^{-5}}{\sqrt{5}} = 724.5\ \mu\text{W}$$

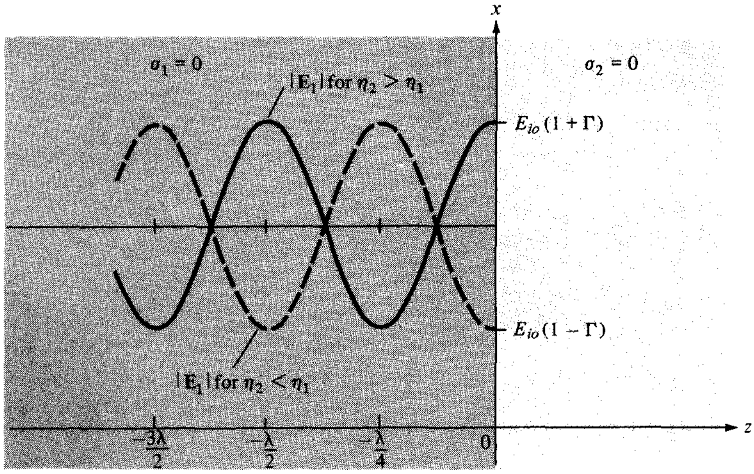


Figure 10.13 Standing waves due to reflection at an interface between two lossless media; $\lambda = 2\pi/\beta_1$.

or

$$|\Gamma| = \frac{s - 1}{s + 1} \tag{10.91}$$

Since $|\Gamma| \leq 1$, it follows that $1 \leq s \leq \infty$. The standing-wave ratio is dimensionless and it is customarily expressed in decibels (dB) as

$$s \text{ in dB} = 20 \log_{10} s \tag{10.92}$$

EXAMPLE 10.8

In free space ($z \leq 0$), a plane wave with

$$\mathbf{H} = 10 \cos (10^8 t - \beta z)\, \mathbf{a}_x \text{ mA/m}$$

is incident normally on a lossless medium ($\varepsilon = 2\varepsilon_o$, $\mu = 8\mu_o$) in region $z \geq 0$. Determine the reflected wave $\mathbf{H}_r$, $\mathbf{E}_r$ and the transmitted wave $\mathbf{H}_t$, $\mathbf{E}_t$.

Solution:

This problem can be solved in two different ways.

Method 1: Consider the problem as illustrated in Figure 10.14. For free space,

$$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

$$\eta_1 = \eta_o = 120\pi$$

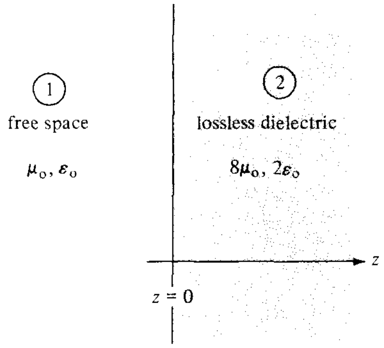


Figure 10.14 For Example 10.8.

For the lossless dielectric medium,

$$\beta_2 = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\cdot(4) = 4\beta_1 = \frac{4}{3}$$

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 2\,\eta_o$$

Given that $\mathbf{H}_i = 10\cos(10^8 t - \beta_1 z)\,\mathbf{a}_x$, we expect that

$$\mathbf{E}_i = E_{io}\cos(10^8 t - \beta_1 z)\,\mathbf{a}_{E_i}$$

where

$$\mathbf{a}_{E_i} = \mathbf{a}_{H_i} \times \mathbf{a}_{k_i} = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$$

and

$$E_{io} = \eta_1 H_{io} = 10\,\eta_o$$

Hence,

$$\mathbf{E}_i = -10\eta_o\cos(10^8 t - \beta_1 z)\,\mathbf{a}_y \text{ mV/m}$$

Now

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_o - \eta_o}{2\eta_o + \eta_o} = \frac{1}{3}$$

$$E_{ro} = \frac{1}{3}E_{io}$$

Thus

$$\mathbf{E}_r = -\frac{10}{3}\eta_o\cos\left(10^8 t + \frac{1}{3}z\right)\mathbf{a}_y \text{ mV/m}$$

from which we easily obtain $\mathbf{H}_r$ as

$$\mathbf{H}_r = -\frac{10}{3}\cos\left(10^8 t + \frac{1}{3}z\right)\mathbf{a}_x \text{ mA/m}$$

Similarly,

$$\frac{E_{to}}{E_{io}} = \tau = 1 + \Gamma = \frac{4}{3} \qquad \text{or} \qquad E_{to} = \frac{4}{3}E_{io}$$

Thus

$$\mathbf{E}_t = E_{to}\cos(10^8 t - \beta_2 z)\,\mathbf{a}_{E_t}$$

where $\mathbf{a}_{E_t} = \mathbf{a}_{E_i} = -\mathbf{a}_y$. Hence,

$$\mathbf{E}_t = -\frac{40}{3}\eta_o\cos\left(10^8 t - \frac{4}{3}z\right)\mathbf{a}_y \text{ mV/m}$$

from which we obtain

$$\mathbf{H}_t = \frac{20}{3}\cos\left(10^8 t - \frac{4}{3}z\right)\mathbf{a}_x \text{ mA/m}$$

Method 2: Alternatively, we can obtain $\mathbf{H}_r$ and $\mathbf{H}_t$ directly from $\mathbf{H}_i$ using

$$\frac{H_{ro}}{H_{io}} = -\Gamma \qquad \text{and} \qquad \frac{H_{to}}{H_{io}} = \tau\frac{\eta_1}{\eta_2}$$

Thus

$$H_{ro} = -\frac{1}{3}H_{io} = -\frac{10}{3}$$

$$H_{to} = \frac{4}{3}\frac{\eta_o}{2\eta_o}\cdot H_{io} = \frac{2}{3}H_{io} = \frac{20}{3}$$

and

$$\mathbf{H}_r = -\frac{10}{3}\cos(10^8 t + \beta_1 z)\,\mathbf{a}_x \text{ mA/m}$$

$$\mathbf{H}_t = \frac{20}{3}\cos(10^8 t - \beta_2 z)\,\mathbf{a}_x \text{ mA/m}$$

as previously obtained.

Notice that the boundary conditions at $z = 0$, namely,

$$\mathbf{E}_i(0) + \mathbf{E}_r(0) = \mathbf{E}_t(0) = -\frac{40}{3}\eta_o\cos(10^8 t)\,\mathbf{a}_y$$

and

$$\mathbf{H}_i(0) + \mathbf{H}_r(0) = \mathbf{H}_t(0) = \frac{20}{3}\cos(10^8 t)\,\mathbf{a}_x$$

are satisfied. These boundary conditions can always be used to cross-check **E** and **H**.

PRACTICE EXERCISE 10.8

A 5-GHz uniform plane wave $\mathbf{E}_{is} = 10\,e^{-j\beta z}\,\mathbf{a}_x$ V/m in free space is incident normally on a large plane, lossless dielectric slab ($z > 0$) having $\varepsilon = 4\varepsilon_o$, $\mu = \mu_o$. Find the reflected wave $\mathbf{E}_{rs}$ and the transmitted wave $\mathbf{E}_{ts}$.

Answer: $-3.333\exp(j\beta_1 z)\,\mathbf{a}_x$ V/m, $6.667\exp(-j\beta_2 z)\,\mathbf{a}_x$ V/m where $\beta_2 = 2\beta_1 = 200\pi/3$.

EXAMPLE 10.9

Given a uniform plane wave in air as

$$\mathbf{E}_i = 40\cos(\omega t - \beta z)\,\mathbf{a}_x + 30\sin(\omega t - \beta z)\,\mathbf{a}_y \text{ V/m}$$

(a) Find $\mathbf{H}_i$.

(b) If the wave encounters a perfectly conducting plate normal to the z axis at $z = 0$, find the reflected wave $\mathbf{E}_r$ and $\mathbf{H}_r$.

(c) What are the total **E** and **H** fields for $z \le 0$?

(d) Calculate the time-average Poynting vectors for $z \le 0$ and $z \ge 0$.

Solution:

(a) This is similar to the problem in Example 10.3. We may treat the wave as consisting of two waves $\mathbf{E}_{i1}$ and $\mathbf{E}_{i2}$, where

$$\mathbf{E}_{i1} = 40\cos(\omega t - \beta z)\,\mathbf{a}_x, \qquad \mathbf{E}_{i2} = 30\sin(\omega t - \beta z)\,\mathbf{a}_y$$

At atmospheric pressure, air has $\varepsilon_r = 1.0006 \simeq 1$. Thus air may be regarded as free space. Let $\mathbf{H}_i = \mathbf{H}_{i1} + \mathbf{H}_{i2}$.

$$\mathbf{H}_{i1} = H_{i1o}\cos(\omega t - \beta z)\,\mathbf{a}_{H_1}$$

where

$$H_{i1o} = \frac{E_{i1o}}{\eta_o} = \frac{40}{120\pi} = \frac{1}{3\pi}$$

$$\mathbf{a}_{H_1} = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

Hence

$$\mathbf{H}_{i1} = \frac{1}{3\pi}\cos(\omega t - \beta z)\,\mathbf{a}_y$$

Similarly,

$$\mathbf{H}_{i2} = H_{i2o}\sin(\omega t - \beta z)\,\mathbf{a}_{H_2}$$

where

$$H_{i2o} = \frac{E_{i2o}}{\eta_o} = \frac{30}{120\pi} = \frac{1}{4\pi}$$

$$\mathbf{a}_{H_2} = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

Hence

$$\mathbf{H}_{i2} = -\frac{1}{4\pi}\sin(\omega t - \beta z)\,\mathbf{a}_x$$

and

$$\mathbf{H}_i = \mathbf{H}_{i1} + \mathbf{H}_{i2}$$
$$= -\frac{1}{4\pi}\sin(\omega t - \beta z)\,\mathbf{a}_x + \frac{1}{3\pi}\cos(\omega t - \beta z)\,\mathbf{a}_y \text{ mA/m}$$

This problem can also be solved using Method 2 of Example 10.3.

(b) Since medium 2 is perfectly conducting,

$$\frac{\sigma_2}{\omega\varepsilon_2} \gg 1 \quad \rightarrow \quad \eta_2 \ll \eta_1$$

that is,

$$\Gamma \simeq -1, \qquad \tau = 0$$

showing that the incident **E** and **H** fields are totally reflected.

$$E_{ro} = \Gamma\, E_{io} = -E_{io}$$

Hence,

$$\mathbf{E}_r = -40\cos(\omega t + \beta z)\,\mathbf{a}_x - 30\sin(\omega t + \beta z)\,\mathbf{a}_y \text{ V/m}$$

$\mathbf{H}_r$ can be found from $\mathbf{E}_r$ just as we did in part (a) of this example or by using Method 2 of the last example starting with $\mathbf{H}_i$. Whichever approach is taken, we obtain

$$\mathbf{H}_r = \frac{1}{3\pi}\cos(\omega t + \beta z)\,\mathbf{a}_y - \frac{1}{4\pi}\sin(\omega t + \beta z)\mathbf{a}_x \text{ A/m}$$

(c) The total fields in air

$$\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r \quad \text{and} \quad \mathbf{H}_1 = \mathbf{H}_i + \mathbf{H}_r$$

can be shown to be standing wave. The total fields in the conductor are

$$\mathbf{E}_2 = \mathbf{E}_t = 0, \qquad \mathbf{H}_2 = \mathbf{H}_t = 0.$$

(d) For $z \leq 0$,

$$\begin{aligned}\mathcal{P}_{1\text{ave}} &= \frac{|\mathbf{E}_{1s}|^2}{2\eta_1}\mathbf{a}_k = \frac{1}{2\eta_o}[E_{io}^2\mathbf{a}_z - E_{ro}^2\mathbf{a}_z] \\ &= \frac{1}{240\pi}[(40^2 + 30^2)\mathbf{a}_z - (40^2 + 30^2)\mathbf{a}_z] \\ &= 0\end{aligned}$$

For $z \geq 0$,

$$\mathcal{P}_{2\text{ave}} = \frac{|\mathbf{E}_{2S}|^2}{2\eta_2}\mathbf{a}_k = \frac{E_{to}^2}{2\eta_2}\mathbf{a}_z = 0$$

because the whole incident power is reflected.

PRACTICE EXERCISE 10.9

The plane wave $\mathbf{E} = 50 \sin(\omega t - 5x)\,\mathbf{a}_y$ V/m in a lossless medium ($\mu = 4\mu_o$, $\varepsilon = \varepsilon_o$) encounters a lossy medium ($\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, $\sigma = 0.1$ mhos/m) normal to the x-axis at $x = 0$. Find

(a) Γ, τ, and s

(b) $\mathbf{E}_r$ and $\mathbf{H}_r$

(c) $\mathbf{E}_t$ and $\mathbf{H}_t$

(d) The time-average Poynting vectors in both regions

Answer: (a) $0.8186\,\underline{/171.1^\circ}$, $0.2295\,\underline{/33.56^\circ}$, 10.025, (b) $40.93 \sin(\omega t + 5x + 171.9^\circ)\,\mathbf{a}_y$ V/m, $-54.3 \sin(\omega t + 5x + 171.9^\circ\,\mathbf{a}_z$ mA/m, (c) $11.47\,e^{-6.021x}\sin(\omega t - 7.826x + 33.56^\circ)\,\mathbf{a}_y$ V/m, $120.2\,e^{-6.021x}\sin(\omega t - 7.826x - 4.01^\circ)\,\mathbf{a}_z$ mA/m, (d) $0.5469\,\mathbf{a}_x$ W/m^2, $0.5469 \exp(-12.04x)\,\mathbf{a}_x$ W/m^2.

EXAMPLE 10.10 An EM wave travels in free space with the electric field component

$$\mathbf{E}_s = 100\, e^{j(0.866y+0.5z)}\, \mathbf{a}_x \text{ V/m}$$

Determine

(a) ω and λ

(b) The magnetic field component

(c) The time average power in the wave

Solution:

(a) Comparing the given **E** with

$$\mathbf{E}_s = \mathbf{E}_o\, e^{j\mathbf{k}\cdot\mathbf{r}} = E_o e^{j(k_x x + k_y y + k_z z)}\, \mathbf{a}_x$$

it is clear that

$$k_x = 0, \qquad k_y = 0.866, \qquad k_z = 0.5$$

Thus

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{(0.866)^2 + (0.5)^2} = 1$$

But in free space,

$$k = \beta = \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Hence,

$$\omega = kc = 3 \times 10^8 \text{ rad/s}$$

$$\lambda = \frac{2\pi}{k} = 2\pi = 6.283 \text{ m}$$

(b) From eq. (10.96), the corresponding magnetic field is given by

$$\begin{aligned}\mathbf{H}_s &= \frac{1}{\mu\omega}\mathbf{k} \times \mathbf{E}_s \\ &= \frac{(0.866\mathbf{a}_y + 0.5\mathbf{a}_z)}{4\pi \times 10^{-7} \times 3 \times 10^8} \times 100\, \mathbf{a}_x e^{j\mathbf{k}\cdot\mathbf{r}}\end{aligned}$$

or

$$\mathbf{H}_s = (1.33\, \mathbf{a}_y - 2.3\, \mathbf{a}_z)\, e^{j(0.866y+0.5z)} \text{ mA/m}$$

(c) The time average power is

$$\begin{aligned}\mathcal{P}_{ave} &= \frac{1}{2}\text{Re}\,(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{E_o^2}{2\eta}\mathbf{a}_k \\ &= \frac{(100)^2}{2(120\pi)}(0.866\, \mathbf{a}_y + 0.5\, \mathbf{a}_z) \\ &= 11.49\, \mathbf{a}_y + 6.631\, \mathbf{a}_z \text{ W/m}^2\end{aligned}$$

PRACTICE EXERCISE 10.10

Rework Example 10.10 if

$$\mathbf{E} = (10\,\mathbf{a}_y + 5\mathbf{a}_z)\cos(\omega t + 2y - 4z)\text{ V/m}$$

in free space.

Answer: (a) 1.342×10^9 rad/s, 1.405 m, (b) $-29.66\cos(1.342 \times 10^9 t + 2y - 4z)\,\mathbf{a}_x$ mA/m, (c) $-0.07415\,\mathbf{a}_y + 0.1489\,\mathbf{a}_z$ W/m^2.

EXAMPLE 10.11

A uniform plane wave in air with

$$\mathbf{E} = 8\cos(\omega t - 4x - 3z)\,\mathbf{a}_y\text{ V/m}$$

is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1.0$, $\varepsilon_r = 2.5$, $\sigma = 0$. Find

(a) The polarization of the wave
(b) The angle of incidence
(c) The reflected **E** field
(d) The transmitted **H** field

Solution:

(a) From the incident **E** field, it is evident that the propagation vector is

$$\mathbf{k}_i = 4\mathbf{a}_x + 3\mathbf{a}_z \rightarrow k_i = 5 = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c}$$

Hence,

$$\omega = 5c = 15 \times 10^8\text{ rad/s.}$$

A unit vector normal to the interface ($z = 0$) is $\mathbf{a}_z$. The plane containing **k** and $\mathbf{a}_z$ is y = constant, which is the xz-plane, the plane of incidence. Since $\mathbf{E}_i$ is normal to this plane, we have perpendicular polarization (similar to Figure 10.17).

(b) The propagation vectors are illustrated in Figure 10.18 where it is clear that

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{4}{3} \rightarrow \theta_i = 53.13°$$

Alternatively, without Figure 10.18, we can obtain θ_i from the fact that θ_i is the angle between **k** and $\mathbf{a}_n$, that is,

$$\cos\theta_i = \mathbf{a}_k \cdot \mathbf{a}_n = \left(\frac{4\mathbf{a}_x + 3\mathbf{a}_z}{5}\right)\cdot\mathbf{a}_z = \frac{3}{5}$$

or

$$\theta_i = 53.13°$$

(c) An easy way to find $\mathbf{E}_r$ is to use eq. (10.116a) because we have noticed that this problem is similar to that considered in Section 10.9(b). Suppose we are not aware of this. Let

$$\mathbf{E}_r = E_{ro} \cos(\omega t - \mathbf{k}_r \cdot \mathbf{r})\, \mathbf{a}_y$$

which is similar to form to the given $\mathbf{E}_i$. The unit vector $\mathbf{a}_y$ is chosen in view of the fact that the tangential component of $\mathbf{E}$ must be continuous at the interface. From Figure 10.18,

$$\mathbf{k}_r = k_{rx}\, \mathbf{a}_x - k_{rz}\, \mathbf{a}_z$$

where

$$k_{rx} = k_r \sin\theta_r, \qquad k_{rz} = k_r \cos\theta_r$$

But $\theta_r = \theta_i$ and $k_r = k_i = 5$ because both k_r and k_i are in the same medium. Hence,

$$k_r = 4\mathbf{a}_x - 3\mathbf{a}_z$$

To find E_{ro}, we need θ_t. From Snell's law

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{\sin 53.13^\circ}{\sqrt{2.5}}$$

or

$$\theta_t = 30.39^\circ$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

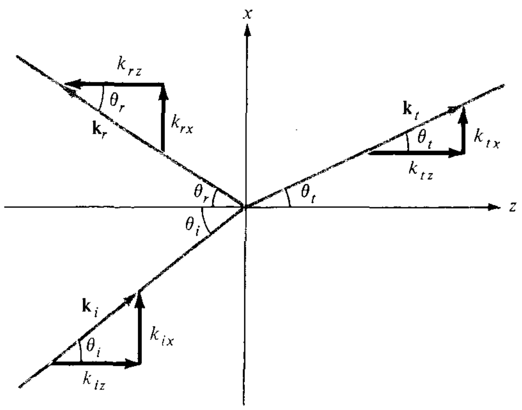


Figure 10.18 Propagation vectors of Example 10.11.

where $\eta_1 = \eta_o = 377$, $\quad \eta_2 = \sqrt{\dfrac{\mu_o \mu_{r_2}}{\varepsilon_o \varepsilon_{r_2}}} = \dfrac{377}{\sqrt{2.5}} = 238.4$

$$\Gamma_\perp = \frac{238.4 \cos 35.13° - 377 \cos 30.39°}{238.4 \cos 53.13° + 377 \cos 30.39°} = -0.389$$

Hence,

$$E_{ro} = \Gamma_\perp E_{io} = -0.389\,(8) = -3.112$$

and

$$\mathbf{E}_r = -3.112 \cos (15 \times 10^8 t - 4x + 3z)\, \mathbf{a}_y \text{ V/m}$$

(d) Similarly, let the transmitted electric field be

$$\mathbf{E}_t = E_{to} \cos (\omega t - \mathbf{k}_t \cdot \mathbf{r})\, \mathbf{a}_y$$

where

$$\begin{aligned} k_t = \beta_2 &= \omega \sqrt{\mu_2 \varepsilon_2} = \frac{\omega}{c} \sqrt{\mu_{r_2} \varepsilon_{r_2}} \\ &= \frac{15 \times 10^8}{3 \times 10^8} \sqrt{1 \times 2.5} = 7.906 \end{aligned}$$

From Figure 10.18,

$$k_{tx} = k_t \sin \theta_t = 4$$

$$k_{tz} = k_t \cos \theta_t = 6.819$$

or

$$\mathbf{k}_t = 4\mathbf{a}_x + 6.819\, \mathbf{a}_z$$

Notice that $k_{ix} = k_{rx} = k_{tx}$ as expected.

$$\begin{aligned} \tau_\perp = \frac{E_{to}}{E_{io}} &= \frac{2\, \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \\ &= \frac{2 \times 238.4 \cos 53.13°}{238.4 \cos 53.13° + 377 \cos 30.39°} \\ &= 0.611 \end{aligned}$$

The same result could be obtained from the relation $\tau_\perp = 1 + \Gamma_\perp$. Hence,

$$E_{to} = \tau_\perp E_{io} = 0.611 \times 8 = 4.888$$

$$\mathbf{E}_t = 4.888 \cos (15 \times 10^8 t - 4x - 6.819z)\, \mathbf{a}_y$$

From $\mathbf{E}_t$, $\mathbf{H}_t$ is easily obtained as

$$\mathbf{H}_t = \frac{1}{\mu_2\omega}\mathbf{k}_t \times \mathbf{E}_t = \frac{\mathbf{a}_{k_t} \times \mathbf{E}_t}{\eta_2}$$

$$= \frac{4\mathbf{a}_x + 6.819\,\mathbf{a}_z}{7.906\,(238.4)} \times 4.888\,\mathbf{a}_y \cos(\omega t - \mathbf{k}\cdot\mathbf{r})$$

$$\mathbf{H}_t = (-17.69\,\mathbf{a}_x + 10.37\,\mathbf{a}_z)\cos(15 \times 10^8 t - 4x - 6.819z)\ \text{mA/m}.$$

PRACTICE EXERCISE 10.11

If the plane wave of Practice Exercise 10.10 is incident on a dielectric medium having $\sigma = 0$, $\varepsilon = 4\varepsilon_o$, $\mu = \mu_o$ and occupying $z \geq 0$, calculate

(a) The angles of incidence, reflection, and transmission
(b) The reflection and transmission coefficients
(c) The total **E** field in free space
(d) The total **E** field in the dielectric
(e) The Brewster angle.

Answer: (a) 26.56°, 26.56°, 12.92°, (b) −0.295, 0.647, (c) $(10\,\mathbf{a}_y + 5\mathbf{a}_z)\cos(\omega t + 2y - 4z) + (-2.946\mathbf{a}_y + 1.473\mathbf{a}_z)\cos(\omega t + 2y + 4z)$ V/m, (d) $(7.055\mathbf{a}_y + 1.618\mathbf{a}_z)\cos(\omega t + 2y - 8.718z)$ V/m, (e) 63.43°.

SUMMARY

1. The wave equation is of the form

$$\frac{\partial^2 \Phi}{\partial t^2} - u^2\frac{\partial^2 \Phi}{\partial z^2} = 0$$

with the solution

$$\Phi = A\sin(\omega t - \beta z)$$

where u = wave velocity, A = wave amplitude, ω = angular frequency ($=2\pi f$), and β = phase constant. Also, $\beta = \omega/u = 2\pi/\lambda$ or $u = f\lambda = \lambda/T$, where λ = wavelength and T = period.

2. In a lossy, charge-free medium, the wave equation based on Maxwell's equations is of the form

$$\nabla^2 \mathbf{A}_s - \gamma^2 \mathbf{A}_s = 0$$

where $\mathbf{A}_s$ is either $\mathbf{E}_s$ or $\mathbf{H}_s$ and $\gamma = \alpha + j\beta$ is the propagation constant. If we assume $\mathbf{E}_s = E_{xs}(z)\,\mathbf{a}_x$, we obtain EM waves of the form

$$\mathbf{E}(z, t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\,\mathbf{a}_x$$

$$\mathbf{H}(z, t) = H_o e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\,\mathbf{a}_y$$

where α = attenuation constant, β = phase constant, $\eta = |\eta|\angle\theta_\eta$ = intrinsic impedance of the medium. The reciprocal of α is the skin depth ($\delta = 1/\alpha$). The relationship between β, ω, and λ as stated above remain valid for EM waves.

3. Wave propagation in other types of media can be derived from that for lossy media as special cases. For free space, set $\sigma = 0$, $\varepsilon = \varepsilon_o$, $\mu = \mu_o$; for lossless dielectric media, set $\sigma = 0$, $\varepsilon = \varepsilon_o\varepsilon_r$, and $\mu = \mu_o\mu_r$; and for good conductors, set $\sigma \simeq \infty$, $\varepsilon = \varepsilon_o$, $\mu = \mu_o$, or $\sigma/\omega\varepsilon \to 0$.
4. A medium is classified as lossy dielectric, lossless dielectric or good conductor depending on its loss tangent given by

$$\tan\theta = \frac{|\mathbf{J}_s|}{|\mathbf{J}_{d_s}|} = \frac{\sigma}{\omega\varepsilon} = \frac{\varepsilon''}{\varepsilon'}$$

where $\varepsilon_c = \varepsilon' - j\varepsilon''$ is the complex permittivity of the medium. For lossless dielectrics $\tan\theta \ll 1$, for good conductors $\tan\theta \gg 1$, and for lossy dielectrics $\tan\theta$ is of the order of unity.

5. In a good conductor, the fields tend to concentrate within the initial distance δ from the conductor surface. This phenomenon is called skin effect. For a conductor of width w and length ℓ, the effective or ac resistance is

$$R_{\text{ac}} = \frac{\ell}{\sigma w \delta}$$

where δ is the skin depth.

6. The Poynting vector, $\mathcal{P}$, is the power-flow vector whose direction is the same as the direction of wave propagation and magnitude the same as the amount of power flowing through a unit area normal to its direction.

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}, \qquad \mathcal{P}_{\text{ave}} = 1/2 \,\text{Re}\,(\mathbf{E}_s \times \mathbf{H}_s^*)$$

7. If a plane wave is incident normally from medium 1 to medium 2, the reflection coefficient Γ and transmission coefficient τ are given by

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \qquad \tau = \frac{E_{to}}{E_{io}} = 1 + \Gamma$$

The standing wave ratio, s, is defined as

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

8. For oblique incidence from lossless medium 1 to lossless medium 2, we have the Fresnel coefficients as

$$\Gamma_{\|} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}, \qquad \tau_{\|} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

for parallel polarization and

$$\Gamma_\perp = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \qquad \tau_\perp = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

for perpendicular polarization. As in optics,

$$\theta_r = \theta_i$$

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{\beta_1}{\beta_2} = \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}}$$

Total transmission or no reflection ($\Gamma = 0$) occurs when the angle of incidence θ_i is equal to the Brewster angle.

REVIEW QUESTIONS

10.1 Which of these is not a correct form of the wave $E_x = \cos(\omega t - \beta z)$?

(a) $\cos(\beta z - \omega t)$
(b) $\sin(\beta z - \omega t - \pi/2)$
(c) $\cos\left(\frac{2\pi t}{T} - \frac{2\pi z}{\lambda}\right)$
(d) $\text{Re}\,(e^{j(\omega t - \beta z)})$
(e) $\cos \beta(z - ut)$

10.2 Identify which of these functions do not satisfy the wave equation:

(a) $50e^{j\omega(t-3z)}$
(b) $\sin \omega(10z + 5t)$
(c) $(x + 2t)^2$
(d) $\cos^2(y + 5t)$
(e) $\sin x \cos t$
(f) $\cos(5y + 2x)$

10.3 Which of the following statements is not true of waves in general?

(a) It may be a function of time only.
(b) It may be sinusoidal or cosinusoidal.
(c) It must be a function of time and space.
(d) For practical reasons, it must be finite in extent.

10.4 The electric field component of a wave in free space is given by $\mathbf{E} = 10 \cos(10^7 t + kz)\,\mathbf{a}_y$ V/m. It can be inferred that

(a) The wave propagates along $\mathbf{a}_y$.
(b) The wavelength $\lambda = 188.5$ m.

(c) The wave amplitude is 10 V/m.
(d) The wave number $k = 0.33$ rad/m.
(e) The wave attenuates as it travels.

10.5 Given that $\mathbf{H} = 0.5\, e^{-0.1x} \sin(10^6 t - 2x)\, \mathbf{a}_z$ A/m, which of these statements are incorrect?

(a) $\alpha = 0.1$ Np/m
(b) $\beta = -2$ rad/m
(c) $\omega = 10^6$ rad/s
(d) The wave travels along $\mathbf{a}_x$.
(e) The wave is polarized in the z-direction.
(f) The period of the wave is 1 μs.

10.6 What is the major factor for determining whether a medium is free space, lossless dielectric, lossy dielectric, or good conductor?

(a) Attenuation constant
(b) Constitutive parameters (σ, ε, μ)
(c) Loss tangent
(d) Reflection coefficient

10.7 In a certain medium, $\mathbf{E} = 10 \cos(10^8 t - 3y)\, \mathbf{a}_x$ V/m. What type of medium is it?

(a) Free space
(b) Perfect dielectric
(c) Lossless dielectric
(d) Perfect conductor

10.8 Electromagnetic waves travel faster in conductors than in dielectrics.

(a) True
(b) False

10.9 In a good conductor, **E** and **H** are in time phase.

(a) True
(b) False

10.10 The Poynting vector physically denotes the power density leaving or entering a given volume in a time-varying field.

(a) True
(b) False

Answers: 10.1b, 10.2d,f, 10.3a, 10.4b,c, 10.5b,f, 10.6c, 10.7c, 10.8b, 10.9b, 10.10a.

PROBLEMS

10.1 An EM wave propagating in a certain medium is described by

$$\mathbf{E} = 25 \sin (2\pi \times 10^6 t - 6x)\, \mathbf{a}_z \text{ V/m}$$

(a) Determine the direction of wave propagation.
(b) Compute the period T, the wavelength λ, and the velocity u.
(c) Sketch the wave at $t = 0, T/8, T/4, T/2$.

10.2 (a) Derive eqs. (10.23) and (10.24) from eqs. (10.18) and (10.20).
(b) Using eq. (10.29) in conjunction with Maxwell's equations, show that

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From part (b), derive eqs. (10.32) and (10.33).

10.3 At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\varepsilon_o$, $\mu = 2.1\mu_o$, and $\sigma = 0.08$ S/m. If $\mathbf{E}_s = 6e^{-\gamma x}\, \mathbf{a}_z$ V/m, compute: (a) γ, (b) λ, (c) u, (d) η, (e) $\mathbf{H}_s$.

10.4 A lossy material has $\mu = 5\mu_o$, $\varepsilon = 2\varepsilon_o$. If at 5 MHz, the phase constant is 10 rad/m, calculate

(a) The loss tangent
(b) The conductivity of the material
(c) The complex permittivity
(d) The attenuation constant
(e) The intrinsic impedance

***10.5** A nonmagnetic medium has an intrinsic impedance $240 \angle 30^\circ\ \Omega$. Find its

(a) Loss tangent
(b) Dielectric constant
(c) Complex permittivity
(d) Attenuation constant at 1 MHz

10.6 The amplitude of a wave traveling through a lossy nonmagnetic medium reduces by 18% every meter. If the wave operates at 10 MHz and the electric field leads the magnetic field by 24°, calculate: (a) the propagation constant, (b) the wavelength, (c) the skin depth, (d) the conductivity of the medium.

10.7 Sea water plays a vital role in the study of submarine communications. Assuming that for sea water, $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$, and $f = 100$ MHz, calculate: (a) the phase velocity, (b) the wavelength, (c) the skin depth, (d) the intrinsic impedance.

10.8 In a certain medium with $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$,

$$\mathbf{H} = 12e^{-0.1y} \sin (\pi \times 10^8 t - \beta y)\, \mathbf{a}_x \text{ A/m}$$

find: (a) the wave period T, (b) the wavelength λ, (c) the electric field $\mathbf{E}$, (d) the phase difference between $\mathbf{E}$ and $\mathbf{H}$.

10.9 In a medium,

$$\mathbf{E} = 16e^{-0.05x} \sin (2 \times 10^8 t - 2x) \, \mathbf{a}_z \text{ V/m}$$

find: (a) the propagation constant, (b) the wavelength, (c) the speed of the wave, (d) the skin depth.

10.10 A uniform wave in air has

$$\mathbf{E} = 10 \cos (2\pi \times 10^6 t - \beta z) \, \mathbf{a}_y$$

(a) Calculate β and λ.

(b) Sketch the wave at $z = 0, \lambda/4$.

(c) Find **H**.

10.11 The magnetic field component of an EM wave propagating through a nonmagnetic medium ($\mu = \mu_o$) is

$$\mathbf{H} = 25 \sin (2 \times 10^8 t + 6x) \, \mathbf{a}_y \text{ mA/m}$$

Determine:

(a) The direction of wave propagation.

(b) The permittivity of the medium.

(c) The electric field intensity.

10.12 If $\mathbf{H} = 10 \sin (\omega t - 4z)\mathbf{a}_x$ mA/m in a material for which $\sigma = 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, calculate ω, λ, and $\mathbf{J}_d$.

10.13 A manufacturer produces a ferrite material with $\mu = 750\mu_o$, $\varepsilon = 5\varepsilon_o$, and $\sigma = 10^{-6}$ S/m at 10 MHz.

(a) Would you classify the material as lossless, lossy, or conducting?

(b) Calculate β and λ.

(c) Determine the phase difference between two points separated by 2 m.

(d) Find the intrinsic impedance.

***10.14** By assuming the time-dependent fields $\mathbf{E} = \mathbf{E}_o e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ and $\mathbf{H} = \mathbf{H}_o e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ where $\mathbf{k} = k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z$ is the wave number vector and $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the radius vector, show that $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ can be expressed as $\mathbf{k} \times \mathbf{E} = \mu\omega\mathbf{H}$ and deduce $\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$.

10.15 Assume the same fields as in Problem 10.14 and show that Maxwell's equations in a source-free region can be written as

$$\mathbf{k} \cdot \mathbf{E} = 0$$

$$\mathbf{k} \cdot \mathbf{H} = 0$$

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}$$

From these equations deduce

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H \qquad \text{and} \qquad \mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$$

10.16 The magnetic field component of a plane wave in a lossless dielectric is

$$\mathbf{H} = 30 \sin (2\pi \times 10^8 t - 5x)\, \mathbf{a}_z \text{ mA/m}$$

(a) If $\mu_r = 1$, find ε_r.
(b) Calculate the wavelength and wave velocity.
(c) Determine the wave impedance.
(d) Determine the polarization of the wave.
(e) Find the corresponding electric field component.
(f) Find the displacement current density.

10.17 In a nonmagnetic medium,

$$\mathbf{E} = 50 \cos (10^9 t - 8x)\, \mathbf{a}_y + 40 \sin (10^9 t - 8x)\, \mathbf{a}_z \text{ V/m}$$

find the dielectric constant ε_r and the corresponding **H**.

10.18 In a certain medium

$$\mathbf{E} = 10 \cos (2\pi \times 10^7 t - \beta x)(\mathbf{a}_y + \mathbf{a}_z) \text{ V/m}$$

If $\mu = 50\mu_o$, $\varepsilon = 2\varepsilon_o$, and $\sigma = 0$, find β and **H**.

10.19 Which of the following media may be treated as conducting at 8 MHz?

(a) Wet marshy soil ($\varepsilon = 15\varepsilon_o$, $\mu = \mu_o$, $\sigma = 10^{-2}$ S/m)
(b) Intrinsic germanium ($\varepsilon = 16\varepsilon_o$, $\mu = \mu_o$, $\sigma = 0.025$ S/m)
(c) Sea water ($\varepsilon = 81\varepsilon_o$, $\mu = \mu_o$, $\sigma = 25$ S/m)

10.20 Calculate the skin depth and the velocity of propagation for a uniform plane wave at frequency 6 MHz traveling in polyvinylchloride ($\mu_r = 1$, $\varepsilon_r = 4$, $\tan \theta_\eta = 7 \times 10^{-2}$).

10.21 A uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz and its magnitude is reduced by 60% for every 2 m traveled. Find the skin depth and speed of the wave.

10.22 (a) Determine the dc resistance of a round copper wire ($\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$) of radius 1.2 mm and length 600 m.
(b) Find the ac resistance at 100 MHz.
(c) Calculate the approximate frequency where dc and ac resistances are equal.

10.23 A 40-m-long aluminum ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$) pipe with inner and outer radii 9 mm and 12 mm carries a total current of $6 \sin 10^6 \pi t$ A. Find the skin depth and the effective resistance of the pipe.

10.24 Show that in a good conductor, the skin depth δ is always much shorter than the wavelength.

10.25 Brass waveguides are often silver plated to reduce losses. If at least the thickness of silver ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$, $\sigma = 6.1 \times 10^7$ S/m) must be 5δ, find the minimum thickness required for a waveguide operating at 12 GHz.

10.26 A uniform plane wave in a lossy nonmagnetic media has

$$\mathbf{E}_s = (5\mathbf{a}_x + 12\mathbf{a}_y)\mathbf{e}^{-\gamma z}, \gamma = 0.2 + j3.4/\text{m}$$

(a) Compute the magnitude of the wave at $z = 4$ m.

(b) Find the loss in dB suffered by the wave in the interval $0 < z < 3$ m.

(c) Calculate the Poynting vector at $z = 4$, $t = T/8$. Take $\omega = 10^8$ rad/s.

10.27 In a nonmagnetic material,

$$\mathbf{H} = 30 \cos (2\pi \times 10^8 t - 6x)\, \mathbf{a}_y \text{ mA/m}$$

find: (a) the intrinsic impedance, (b) the Poynting vector, (c) the time-average power crossing the surface $x = 1, 0 < y < 2, 0 < z < 3$ m.

***10.28** Show that eqs. (10.67) and (10.68) are equivalent.

10.29 In a transmission line filled with a lossless dielectric ($\varepsilon = 4.5\varepsilon_o$, $\mu = \mu_o$),

$$\mathbf{E} = \frac{40}{\rho} \sin (\omega t - 2z)\, \mathbf{a}_\rho \text{ V/m}$$

find: (a) ω and **H**, (b) the Poynting vector, (c) the total time-average power crossing the surface $z = 1$ m, $2 \text{ mm} < \rho < 3 \text{ mm}$, $0 < \phi < 2\pi$.

10.30 (a) For a normal incidence upon the dielectric–dielectric interface for which $\mu_1 = \mu_2 = \mu_o$, we define R and T as the reflection and transmission coefficients for average powers, i.e., $P_{r,\text{ave}} = RP_{i,\text{ave}}$ and $P_{t,\text{ave}} = TP_{i,\text{ave}}$. Prove that

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad \text{and} \quad T = \frac{4n_1n_2}{(n_1 + n_2)^2}$$

where n_1 and n_2 are the reflective indices of the media.

(b) Determine the ratio n_1/n_2 so that the reflected and the transmitted waves have the same average power.

10.31 The plane wave $\mathbf{E} = 30 \cos(\omega t - z)\mathbf{a}_x$ V/m in air normally hits a lossless medium ($\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$) at $z = 0$. (a) Find Γ, τ, and s. (b) Calculate the reflected electric and magnetic fields.

10.32 A uniform plane wave in air with

$$\mathbf{H} = 4 \sin (\omega t - 5x)\, \mathbf{a}_y \text{ A/m}$$

is normally incident on a plastic region with the parameters $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, and $\sigma = 0$. (a) Obtain the total electric field in air. (b) Calculate the time-average power density in the plastic region. (c) Find the standing wave ratio.

10.33 A plane wave in free space with $\mathbf{E} = 3.6 \cos(\omega t - 3x)\, \mathbf{a}_y$ V/m is incident normally on an interface at $x = 0$. If a lossless medium with $\sigma = 0$, $\varepsilon_r = 12.5$ exits for $x \geq 0$ and the reflected wave has $\mathbf{H}_r = -1.2 \cos(\omega t + 3x)\, \mathbf{a}_z$ mA/m, find μ_2.

10.34 Region 1 is a lossless medium for which $y \geq 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, whereas region 2 is free space, $y \leq 0$. If a plane wave $\mathbf{E} = 5 \cos(10^8 t + \beta y)\, \mathbf{a}_z$ V/m exists in region 1, find: (a) the total electric field component of the wave in region 2, (b) the time-average Poynting vector in region 1, (c) the time-average Poynting vector in region 2.

10.35 A plane wave in free space ($z \leq 0$) is incident normally on a large block of material with $\varepsilon_r = 12$, $\mu_r = 3$, $\sigma = 0$ which occupies $z \geq 0$. If the incident electric field is

$$\mathbf{E} = 30 \cos(\omega t - z)\, \mathbf{a}_y \text{ V/m}$$

find: (a) ω, (b) the standing wave ratio, (c) the reflected magnetic field, (d) the average power density of the transmitted wave.

10.36 A 30-MHz uniform plane wave with

$$\mathbf{H} = 10 \sin(\omega t + \beta x)\, \mathbf{a}_z \text{ mA/m}$$

exists in region $x \geq 0$ having $\sigma = 0$, $\varepsilon = 9\varepsilon_o$, $\mu = 4\mu_o$. At $x = 0$, the wave encounters free space. Determine (a) the polarization of the wave, (b) the phase constant β, (c) the displacement current density in region $x \geq 0$, (d) the reflected and transmitted magnetic fields, and (e) the average power density in each region.

10.37 A uniform plane wave in air is normally incident on an infinite lossless dielectric material having $\varepsilon = 3\varepsilon_o$ and $\mu = \mu_o$. If the incident wave is $\mathbf{E}_i = 10 \cos(\omega t - z)\, \mathbf{a}_y$ V/m, find:

(a) λ and ω of the wave in air and the transmitted wave in the dielectric medium
(b) The incident $\mathbf{H}_i$ field
(c) Γ and τ
(d) The total electric field and the time-average power in both regions

***10.38** A signal in air ($z \geq 0$) with the electric field component

$$\mathbf{E} = 10 \sin(\omega t + 3z)\, \mathbf{a}_x \text{ V/m}$$

hits normally the ocean surface at $z = 0$ as in Figure 10.19. Assuming that the ocean surface is smooth and that $\varepsilon = 80\varepsilon_o$, $\mu = \mu_o$, $\sigma = 4$ mhos/m in ocean, determine

(a) ω
(b) The wavelength of the signal in air
(c) The loss tangent and intrinsic impedance of the ocean
(d) The reflected and transmitted $\mathbf{E}$ field

10.39 Sketch the standing wave in eq. (10.87) at $t = 0$, $T/8$, $T/4$, $3T/8$, $T/2$, and so on, where $T = 2\pi/\omega$.

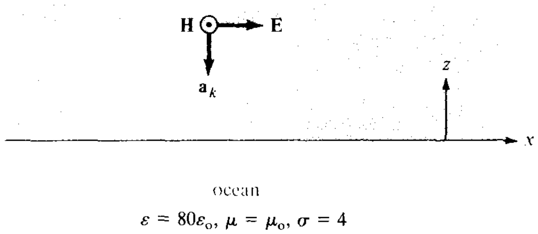


Figure 10.19 For Problem 10.38.

10.40 A uniform plane wave is incident at an angle $\theta_i = 45°$ on a pair of dielectric slabs joined together as shown in Figure 10.20. Determine the angles of transmission θ_{t1} and θ_{t2} in the slabs.

10.41 Show that the field

$$\mathbf{E}_s = 20 \sin(k_x x) \cos(k_y y)\, \mathbf{a}_z$$

where $k_x^2 + k_y^2 = \omega^2 \mu_o \varepsilon_o$, can be represented as the superposition of four propagating plane waves. Find the corresponding $\mathbf{H}_s$.

10.42 Show that for nonmagnetic dielectric media, the reflection and transmission coefficients for oblique incidence become

$$\Gamma_{\|} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}, \qquad \tau_{\|} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)\cos(\theta_t - \theta_i)}$$

$$\Gamma_{\perp} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \qquad \tau_{\perp} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}$$

***10.43** A parallel-polarized wave in air with

$$\mathbf{E} = (8\mathbf{a}_y - 6\mathbf{a}_z)\sin(\omega t - 4y - 3z)\ \text{V/m}$$

impinges a dielectric half-space as shown in Figure 10.21. Find: (a) the incidence angle θ_i, (b) the time average in air ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$), (c) the reflected and transmitted **E** fields.

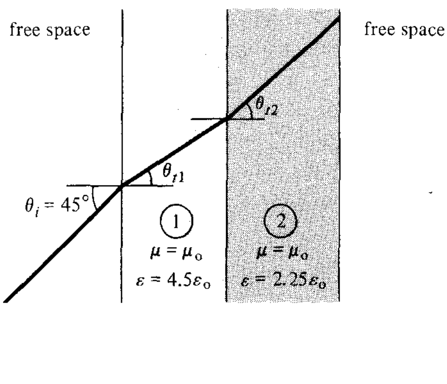


Figure 10.20 For Problem 10.40.

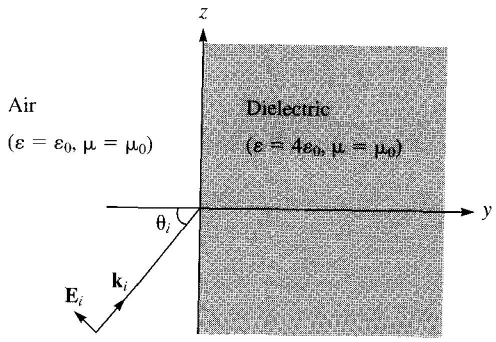


Figure 10.21 For Problem 10.43.

10.44 In a dielectric medium ($\varepsilon = 9\varepsilon_o$, $\mu = \mu_o$), a plane wave with

$$\mathbf{H} = 0.2 \cos (10^9 t - kx - k\sqrt{8}z)\, \mathbf{a}_y \text{ A/m}$$

is incident on an air boundary at $z = 0$, find

(a) θ_r and θ_t

(b) k

(c) The wavelength in the dielectric and air

(d) The incident **E**

(e) The transmitted and reflected **E**

(f) The Brewster angle

***10.45** A plane wave in air with

$$\mathbf{E} = (8\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z) \sin (\omega t + 3x - 4y) \text{ V/m}$$

is incident on a copper slab in $y \geq 0$. Find ω and the reflected wave. Assume copper is a perfect conductor. (*Hint:* Write down the field components in both media and match the boundary conditions.)

10.46 A polarized wave is incident from air to polystyrene with $\mu = \mu_o$, $\varepsilon = 2.6\varepsilon$ at Brewster angle. Determine the transmission angle.

CHAPTER 10

P. E. 10.1 (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2x10^8} = \underline{31.42 \text{ ns}},$$

$$\lambda = uT = 3x10^8 x31.42x10^{-9} = \underline{9.425 \text{ m}}$$

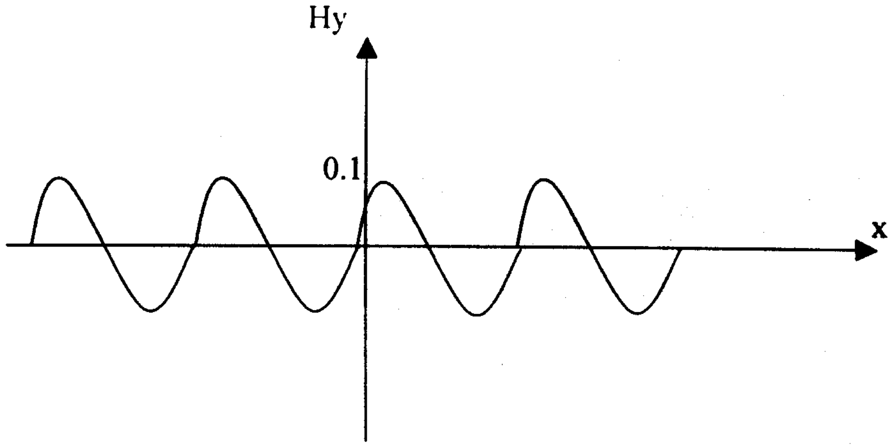
$$k = \beta = 2\pi / \lambda = \underline{0.677 \text{ rad/m}}$$

(b) $t_1 = T/8 = \underline{3.927 \text{ ns}}$

(c)

$$H(t = t_1) = 0.1\cos(2x10^8 \frac{\pi}{8x10^8} - 2x/3)a_y = 0.1\cos(2x/3 - \pi/4)a_y$$

as sketched below.



P. E. 10.2 Let $x_o = \sqrt{1 + (\sigma / \omega\varepsilon)^2}$, then

$$\alpha = \omega \sqrt{\frac{\mu_o \varepsilon_o}{2} \mu_r \varepsilon_r (x_o - 1)} = \frac{\omega}{c} \sqrt{\frac{16}{2}} \sqrt{x_o - 1}$$

or $$\sqrt{x_o - 1} = \frac{\alpha c}{\omega \sqrt{8}} = \frac{1/3x3x10^8}{10^8 \sqrt{8}} = \frac{1}{\sqrt{8}} \longrightarrow x_o = 9/8$$

$$x_o^2 = \frac{81}{64} = 1 + (\sigma / \omega\varepsilon)^2 \longrightarrow \frac{\sigma}{\omega\varepsilon} = 0.5154$$

$$\tan 2\theta_\eta = 0.5154 \longrightarrow \theta_\eta = 13.63^\circ$$

$$\frac{\beta}{\alpha} = \sqrt{\frac{x_o + 1}{x_o - 1}} = \sqrt{17}$$

(a) $\beta = \alpha\sqrt{17} = \frac{\sqrt{17}}{3} =$ 1.374 rad/m

(b) $\frac{\sigma}{\omega\varepsilon} = 0.5154$

(c) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x_o}} = \frac{120\pi\sqrt{2/8}}{\sqrt{9/8}} = 177.72$

$\eta = 177.72\angle 13.63^\circ\,\Omega$

(d) $u = \frac{\omega}{\beta} = \frac{10^8}{1.374} = 7.278x10^7$ m/s

(e) $a_H = a_k x a_E \longrightarrow a_x x a_H = a_z \longrightarrow a_H = a_y$

$$H = \frac{0.5}{177.5} e^{-z/3} \sin(10^8 t - \beta z - 13.63^\circ) a_y = 2.817 e^{-z/3} \sin(10^8 t - \beta z - 13.63^\circ) a_y \text{ mA/m}$$

P. E. 10.3 (a) Along -z direction

(b) $\lambda = \frac{2\pi}{\beta} = 2\pi/2 =$ 3.142 m

$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} =$ 15.92 MHz

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{(1)\varepsilon_r}$$

or $\sqrt{\varepsilon_r} = \beta c/\omega = \frac{3x10^8 x2}{2x10^8} = 6 \longrightarrow \varepsilon_r = 3.6$

(c) $\theta_\eta = 0, |\eta| = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{1/\varepsilon_r} = \frac{120\pi}{6} = 20\pi$

$a_k = a_E x a_H \longrightarrow -a_z = a_x x a_H \longrightarrow a_H = a_x$

$$H = \frac{50}{20\pi}\sin(\omega t + \beta z)a_x = \underline{\underline{795.8\sin(10^8 t + 2z)a_x}} \quad \text{mA/m}$$

P. E. 10.4 (a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{10^9 \pi x 4x \frac{10^{-9}}{36\pi}} = 0.09$$

$$\alpha \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{\omega}{2c}\sqrt{\mu_r\varepsilon_r}\,\frac{\sigma}{\omega\varepsilon} = \frac{10^9\pi}{2x3x10^8}(2)(0.09) = 0.9425$$

$$\beta \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1\right]} = \frac{10^9\pi}{3x10^8}\sqrt{2[2+0.5(0.09)^2]} = 20.965$$

$$E = 30e^{-0.9425y}\cos(10^9\pi t - 20.96y + \pi/4)a_z$$

At t = 2ns, y = 1m,

$$E = 30e^{-0.9425}\cos(2\pi - 20.96 + \pi/4)a_z = \underline{\underline{2.787a_z}} \quad \text{V/m}$$

(b) $\beta y = 10^o = \frac{10\pi}{180}$ rad

or

$$y = \frac{\pi}{18}\frac{1}{\beta} = \frac{\pi}{18x20.905} = \underline{\underline{8.325 \text{ mm}}}$$

(c) $30(0.6) = 30\ e^{-\alpha y}$

$$y = \frac{1}{\alpha}\ln(1/0.6) = \frac{1}{0.9425}\ln\frac{1}{0.6} = \underline{\underline{542 \text{ mm}}}$$

(d)

$$|\eta| \cong \frac{\sqrt{\mu/\varepsilon}}{[1+\frac{1}{4}(0.09)^2]} = \frac{60\pi}{1.002} = 188.11$$

$$2\theta_\eta = \tan^{-1}0.09 \longrightarrow \theta_\eta = 2.571^o$$

$$a_H = a_k x a_E = a_y x a_z = a_x$$

$$H = \frac{30}{188.11} e^{-0.9425y} \cos(10^9 \pi t - 20.96y + \pi/4 - 2.571^o) a_x$$

At y = 2m, t = 5ns,

$$H = (0.1595)(0.1518)\ \cos(-4.5165 rad) a_x = \underline{\underline{-4.71 a_x}} \text{ mA/m}$$

P. E. 10.5

$$I_s = \int_0^w \int_0^\infty J_{xs} dydz = J_{xs}(0) \int_0^w dy \int_0^\infty e^{-z(1+j)\delta} dz = \frac{J_{xs}(0) w\delta}{1+j}$$

$$\underline{\underline{|I_s| = \frac{J_{xs}(0) w\delta}{\sqrt{2}}}}$$

P. E. 10.6 (a)

$$\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu \sigma} = \frac{1.3x10^{-3}}{2}\sqrt{\pi x 10^7 x 4\pi x 10^{-7} x 3.5 x 10^7} = \underline{\underline{24.16}}$$

(b)

$$\frac{R_{ac}}{R_{dc}} = \frac{1.3x10^{-3}}{2}\sqrt{\pi x 2x10^9 x 4\pi x 10^{-7} x 3.5 x 10^7} = \underline{\underline{1080.54}}$$

P. E. 10.7

$$\mathcal{P}_{ave} = \frac{1}{2} \eta H_o^2 a_x$$

(a) Let f(x,z) = x + z – 1 = 0

$$a_n = \frac{\nabla f}{|\nabla f|} = \frac{a_x + a_z}{\sqrt{2}}, \qquad dS = dSa_n$$

$$P_t = \int \mathcal{P}.dS = \mathcal{P}.Sa_n = \frac{1}{2}\eta H_o^2 a_x . \frac{a_x + a_z}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}(120\pi)(0.2)^2(0.1)^2 = 53.31 \text{ mW}$$

(d) $d\mathbf{S} = dydz\mathbf{a}_x, \quad P_t = \int \mathcal{P}.d\mathbf{S} = \frac{1}{2}\eta H_o^2 S$

$$P_t = \frac{1}{2}(120\pi)(0.2)^2\pi(0.05)^2 = \underline{\underline{59.22 \text{ mW}}}$$

P. E. 10.8 $\quad \eta_1 = \eta_o = 120\pi, \eta_2 = \sqrt{\frac{\eta}{\varepsilon}} = \frac{\eta_o}{2}$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 2/3, \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3$$

$$E_{ro} = \Gamma E_{io} = -\frac{10}{3}$$

$$\underline{\underline{E_{rs} = -\frac{10}{3}e^{j\beta_1 z}a_x \text{ V/m}}}$$

where $\beta_1 = \omega / c = 100\pi / 3$.

$$E_{to} = \tau E_{io} = \frac{20}{3}$$

$$\underline{\underline{E_{ts} = \frac{20}{3}e^{-j\beta_2 z}a_x \text{ V/m}}}$$

where $\beta_2 = \omega\sqrt{\varepsilon_r} / c = 2\beta_1 = 200\pi / 3$.

P. E. 10.9

$$\alpha_1 = 0, \ \beta_1 = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\omega}{c} = 5 \longrightarrow \omega = 5c/2 = 7.5x10^8$$

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{0.1}{7.5x10^8 x 4 x \frac{10^{-9}}{36\pi}} = 1.2\pi$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1 + 1.44\pi^2} - 1\right]} = 6.021$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1 + 1.44\pi^2} + 1\right]} =$$ [redacted] 7,826

$$|\eta_2| = \frac{60\pi}{\sqrt[4]{1+1.44\pi^2}} = 95.445, \eta_1 = 120\pi\sqrt{\varepsilon_{r1}} = 754$$

$$\tan 2\theta_{\eta_2} = 1.2\pi \longrightarrow \theta_{\eta_2} = 37.57^o$$

$$\eta_2 = 95.445\angle 37.57^o$$

(a)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.445\angle 37.57^o - 754}{95.445\angle 37.57^o + 754} = \underline{\underline{0.8186\angle 171.08^o}}$$

$$\tau = 1 + \Gamma = \underline{\underline{0.2295\angle 33.56^o}}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.8186}{1-0.8186} = \underline{\underline{10.025}}$$

(b) $E_i = 50\sin(\omega t - 5x)a_y = \text{Im}(E_{is}e^{j\omega t})$, where $E_{is} = 50e^{-j5x}a_y$.

$$E_{ro} = \Gamma E_{io} = 0.8186e^{j171.08^o}(50) = 40.93e^{j171.08^o}$$

$$E_{rs} = 40.93e^{j5x+j171.08^o}a_y$$

$$E_r = \text{Im}(E_{rs}e^{j\omega t}) = \underline{\underline{40.93\sin(\omega t + 5x + 171.1^o)a_y}} \text{ V/m}$$

$$a_H = a_k x a_E = -a_x x a_y = -a_z$$

$$H_r = -\frac{40.93}{754}\sin(\omega t + 5x + 171.1^o)a_z = \underline{\underline{-0.0543\sin(\omega t + 5x + 171.1^o)a_z}} \text{ A/m}$$

(c)

$$E_{to} = \tau E_{io} = 0.229e^{j33.56^o}(50) = 11.475e^{j33.56^o}$$

$$E_{ts} = 11.475e^{-j\beta_2 x + j33.56^o}e^{-\alpha_2 x}a_y$$

$$E_t = \text{Im}(E_{ts}e^{j\omega t}) = \underline{\underline{11.475e^{-6.021x}\sin(\omega t - 7.826x + 33.56^o)a_y}} \text{ V/m}$$

$$a_H = a_k x a_E = a_x x a_y = a_z$$

$$H_t = \frac{11.495}{95.445} e^{-6.021x} \sin(\omega t - 7.826x + 33.56^o - 37.57^o) a_z$$

$$= \underline{\underline{0.1202 e^{-6.021x} \sin(\omega t - 7.826x - 4.01^o) a_z}} \text{ A/m}$$

(d)

$$\mathcal{P}_{1ave} = \frac{E_{io}^2}{2\eta_1} a_x + \frac{E_{ro}^2}{2\eta_1}(-a_x) = \frac{1}{2(240\pi)}[50^2 a_x - 40.93^2 a_x] = \underline{\underline{0.5469 a_x}} \text{ W/m}^2$$

$$\mathcal{P}_{2ave} = \frac{E_{to}^2}{2|\eta_2|} e^{-2\alpha_2 x} \cos\theta_{\eta_2} a_x = \frac{(11.475)^2}{2(95.445)} \cos 37.57^o e^{-2(6.021)x} a_x = \underline{\underline{0.5469 e^{-12.04x} a_x}} \text{ W/m}^2$$

P. E. 10.10 (a)

$$k = -2a_y + 4a_z \longrightarrow k = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\omega = kc = 3x10^8 \sqrt{20} = \underline{\underline{1.342x10^9}} \text{ rad/s},$$

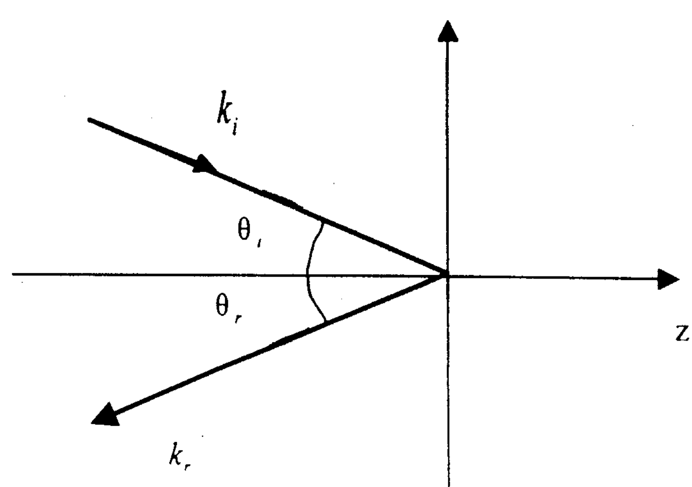
$$\lambda = 2\pi k = \underline{\underline{28.1}}\text{m}$$

(b) $$H = \frac{a_k x E}{\eta_o} = \frac{(-2a_y + 4a_z)}{\sqrt{20}(120\pi)} x(10a_y + 5a_z)\cos(\omega t - k.r)$$

$$= \underline{\underline{-29.66 \cos(1.342x10^9 t + 2y - 4z) a_x}} \text{ mA/m}$$

(c) $$\mathcal{P}_{ave} = \frac{|E_o|^2}{2\eta_o} a_k = \frac{125}{2(120\pi)} \frac{(-2a_y + 4a_z)}{\sqrt{20}} = \underline{\underline{-74.15 a_y + 148.9 a_z}} \text{ W/m}^2$$

P. E. 10.11 (a)



$$\tan\theta_i = \frac{k_{iy}}{k_{iz}} = \frac{2}{4} \longrightarrow \underline{\underline{\theta_i = 26.56 = \theta_r}}$$

$$\sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i = \frac{1}{2}\sin 26.56^o \longrightarrow \underline{\underline{\theta_t = 12.92^o}}$$

(b) $\eta_1 = \eta_o, \eta_2 = \eta_o/2$, **E** is parallel to the plane of incidence. Since $\mu_1 = \mu_2 = \mu_o$, we may use the result of Prob. 10.42, i.e.

$$\Gamma_{\parallel} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\tan(-13.64^o)}{\tan(39.48^o)} = \underline{\underline{-0.2946}}$$

$$\tau_{\parallel} = \frac{2\cos 26.56^o \sin 12.92^o}{\sin 39.48^o \cos(-13.64^o)} = \underline{\underline{0.6474}}$$

(c) $k_r = -\beta_1 \sin\theta_r a_y - \beta_1 \cos\theta_r a_z$. Once k_r is known, E_r is chosen such that

$k_r . E_r = 0$ or $\nabla . E_r = 0$. Let

$$E_r = \pm E_{or}(-\cos\theta_r a_y + \sin\theta_r a_z)\cos(\omega t + \beta_1 \sin\theta_r y + \beta_1 \cos\theta_r z)$$

Only the positive sign will satisfy the boundary conditions. It is evident that

$$E_i = E_{oi}(\cos\theta_i a_y + \sin\theta_i a_z)\cos(\omega t + 2y - 4z)$$

Since $\theta_r = \theta_i$,

$$E_{or}\cos\theta_r = \Gamma_{//}E_{oi}\cos\theta_i = 10\Gamma_{//} = -2.946$$

$$E_{or}\sin\theta_r = \Gamma_{//}E_{oi}\sin\theta_i = 5\Gamma_{//} = -1.473$$

$$\beta_1 \sin\theta_r = 2, \beta_1 \cos\theta_r = 4$$

i.e.

$$E_r = -(2.946a_y - 1.473a_z)\cos(\omega t + 2y + 4z)$$

$$E_1 = E_i + E_r = \underline{\underline{(10a_y + 5a_z)\cos(\omega t + 2y - 4z) + (-2.946a_y + 1.473a_z)\cos(\omega t + 2y + 4z)}}$$

V/m

(d) $k_t = -\beta_2 \sin\theta_t a_y + \beta_2 \cos\theta_t a_z$. Since $k_r \bullet E_r = 0$, let

$$E_t = E_{ot}(\cos\theta_t a_y + \sin\theta_t a_z)\cos(\omega t + \beta_2 y \sin\theta_t - \beta_2 z \cos\theta_t)$$

$$\beta_2 = \omega\sqrt{\mu_2 \varepsilon_2} = \beta_1 \sqrt{\varepsilon_{r2}} = 2\sqrt{20}$$

$$\sin\theta_t = \frac{1}{2}\sin\theta_i = \frac{1}{2\sqrt{5}}, \qquad \cos\theta_t = \frac{\sqrt{9}}{\sqrt{20}}$$

$$\beta_2 \cos\theta_t = 2\sqrt{20}\sqrt{\frac{19}{20}} = 8.718$$

$$E_{ot}\cos\theta_t = \tau_{//} E_{oi}\cos\theta_t = 0.6474\sqrt{125}\sqrt{\frac{19}{20}} = 7.055$$

$$E_{ot}\sin\theta_t = \tau_{//} E_{oi}\sin\theta_t = 0.6474\sqrt{125}\sqrt{\frac{1}{20}} = 1.6185$$

Hence

$$E_2 = E_t = \underline{\underline{(7.055 a_y + 1.6185 a_z)\cos(\omega t + 2y - 8.718z)}} \text{ V/m}$$

(d) $\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 2 \longrightarrow \underline{\underline{\theta_{B//} = 63.43^o}}$

Prob. 10.1 (a) Wave propagates along $+\mathbf{a}_x$.

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi x 10^6} = \underline{\underline{1 \mu s}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{\underline{1.047 \text{ m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi x 10^6}{6} = \underline{\underline{1.047 x 10^6 \text{ m/s}}}$$

(c) At t=0, $E_z = 25\sin(-6x) = -25\sin 6x$

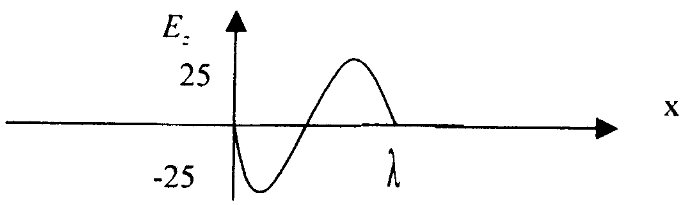
At t=T/8, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{8} - 6x) = 25\sin(\frac{\pi}{4} - 6x)$

At t=T/4, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{4} - 6x) = 25\sin(-6x + 90^o) = 25\cos 6x$

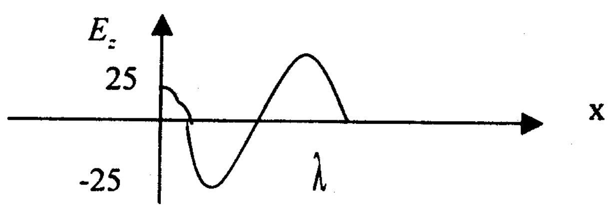
At t=T/2, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{2} - 6x) = 25\sin(-6x + \pi) = 25\sin 6x$

These are sketched below.

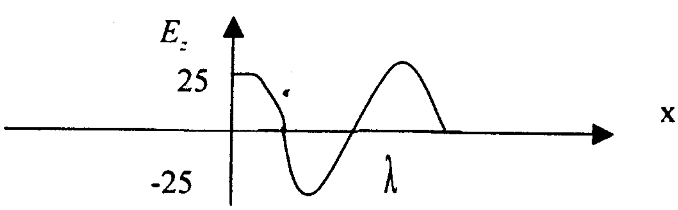
t=0



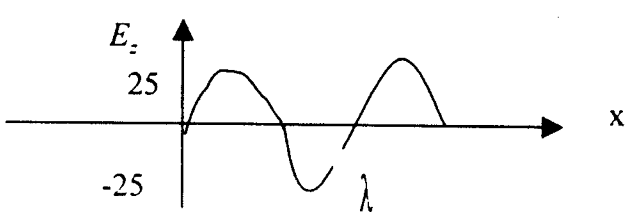
t=T/8



t=T/4



t=T/2



Prob. 10.2 If

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = -\omega^2\mu\varepsilon + j\omega\mu\sigma \quad \text{and}\ \gamma = \alpha + j\beta, \text{ then}$$

$$|\gamma^2| = \sqrt{(\alpha^2 - \beta^2) + 4\alpha^2\beta^2} = \sqrt{(\alpha^2 + \beta^2)^2} = \alpha^2 + \beta^2$$

i.e.

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{(\sigma^2 + \omega^2\varepsilon^2)} \qquad (1)$$

$$Re(\gamma^2) = \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \qquad (2)$$

Subtracting and adding (1) and (2) lead respectively to

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

(b) From eq. (10.25), $E_s(z) = E_o e^{-\gamma z} a_x$.

$$\nabla x E = -j\omega\mu H_s \longrightarrow H_s = \frac{j}{\omega\mu}\nabla x E_s = \frac{j}{\omega\mu}(-\gamma E_o e^{-\gamma z} a_y)$$

But $H_s(z) = H_o e^{-\gamma z} a_y$, hence $H_o = \dfrac{E_o}{\eta} = -\dfrac{j\gamma}{\omega\mu}E_o$

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From (b),

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - j\dfrac{\sigma}{\omega\varepsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}}, \tan 2\theta_\eta = \left(\frac{\omega\varepsilon}{\sigma}\right)^{-1} = \frac{\sigma}{\omega\varepsilon}$$

Prob. 10.3 (a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{8x10^{-2}}{50x10^6 x3.6x\frac{10^{-9}}{36\pi}} = 8$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi x50x10^6}{3x10^8}\sqrt{\frac{2.1x3.6}{2}[\sqrt{65}-1]} = 5.41$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} = 6.129$$

$$\gamma = \alpha + j\beta = \underline{\underline{5.41 + j6.129}} \text{ /m}$$

(b) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.129} = \underline{\underline{1.025}}\text{ m}$$

(c) $$u = \frac{\omega}{\beta} = \frac{2\pi x50x10^6}{6.129} = \underline{\underline{5.125x10^7}} \text{ m/s}$$

(d) $$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi\sqrt{\frac{2.1}{3.6}}}{\sqrt[4]{65}} = 101.4$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 8 \longrightarrow \theta_\eta = 41.44^o$$

$$\eta = \underline{\underline{101.41\angle 41.44^o\,\Omega}}$$

(e) $$H_s = a_k x\frac{E_s}{\eta} = a_x x\frac{6}{\eta}e^{-\gamma z}a_z = -\frac{6}{\eta}e^{-\gamma z}a_y = \underline{\underline{-59.16e^{-j41.44^o}e^{-\gamma z}a_y}} \text{ mA/m}$$

Prob. 10.4 (a) Let $u = \frac{\sigma}{\omega\varepsilon}$ = loss tangent

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+u^2}+1\right]}$$

$$10 = \frac{\omega}{c}\sqrt{\frac{5x2}{2}\left[\sqrt{1+u^2}+1\right]} = \frac{2\pi x5x10^6\sqrt{5}}{3x10^8}\sqrt{\left[\sqrt{1+u^2}+1\right]}$$

which leads to

$$u = \frac{\sigma}{\omega\varepsilon} = \underline{\underline{1823}}$$

(b) $\sigma = \omega\varepsilon u = 2\pi x5x10^6 x1823x\frac{10^{-9}}{36\pi} = \underline{\underline{1.013}}$ S/m

(c) $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\frac{\sigma}{\omega} = 2x\frac{10^{-9}}{36\pi} - j\frac{1.023}{2\pi x5x10^6} = \underline{\underline{1.768x10^{-11} - j3.224x10^{-8}}}$ F/m

d) $\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+u^2}-1}}{\sqrt{\sqrt{1+u^2}+1}} = \sqrt{\frac{1822}{1824}}$

$\alpha = \underline{\underline{9.995}}$ Np/m

(e) $|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+u^2}} = \frac{120\pi\sqrt{\frac{5}{2}}}{\sqrt[4]{1+1823^2}} = 13.96$

$\tan 2\theta_\eta = u = 1823 \longrightarrow \theta_\eta = 44.98^o$

$\eta = \underline{\underline{13.96\angle 44.98^o\,\Omega}}$

Prob. 10.5 (a) $\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60^o = \underline{\underline{1.732}}$

(b) $|\eta| = 240 = \frac{\frac{120\pi}{\sqrt{\varepsilon_r}}}{\sqrt[4]{1+3}} = \frac{120\pi}{\sqrt{2\varepsilon_r}} \longrightarrow \varepsilon_r = \frac{\pi^2}{8} = \underline{\underline{1.234}}$

(c) $\varepsilon_c = \varepsilon(1 - j\frac{\sigma}{\omega\varepsilon}) = 1.234x\frac{10^{-9}}{36\pi}(1 - j1.732) = \underline{\underline{(1.091 - j1.89)x10^{-11}}}$ F/m

(d)

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi x10^6}{3x10^8}\sqrt{\frac{1}{2}\frac{\pi^2}{8}\left[\sqrt{1+3} - 1\right]} = \underline{\underline{0.0164}} \text{ Np/m}$$

Prob. 10.6 (a) $|E| = E_o e^{-\alpha z}$

$$E_o e^{-\alpha(1)} = (1 - 0.18)E_o \longrightarrow e^{-\alpha} = 0.82$$

$$\alpha = \ln\frac{1}{0.82} = 0.1984$$

$$\theta_\eta = 24^o \longrightarrow \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 1.111$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}} = \sqrt{\frac{\sqrt{2.233} - 1}{\sqrt{2.233} + 1}} = 2.247, \quad \beta = 0.4458$$

$$\gamma = \alpha + j\beta = \underline{\underline{0.1984 + j0.4458}} \text{ /m}$$

(b) $\lambda = \frac{2\pi}{\beta} = 2\pi / 0.4458 = \underline{\underline{14.09}}$ m

(c) $\delta = 1/\alpha = \underline{\underline{5.04}}$ m

(d) Since

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}}\sqrt{0.494}, \qquad \mu_r = 1$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{0.494}} = \frac{0.1984x3x10^8}{2\pi x10^7\sqrt{0.494}} = 1.348 \longrightarrow \varepsilon_r = 3.633$$

Since $\frac{\sigma}{\omega\varepsilon} = 1.111$

$$\sigma = \omega\varepsilon_o\varepsilon_r x1.111 = 2\pi x10^7 x\frac{10^{-9}}{36\pi}x3.633x1.111 = \underline{\underline{2.24x10^{-3}}} \text{ S/m}$$

Prob. 10.7

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi x10^5 x81x10^{-9}/36\pi} = \frac{80,000}{9} >> 1$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi x10^5}{2}x4\pi x10^{-7}x4} = 0.4\pi$$

(a) $u = \omega/\beta = \frac{2\pi x10^5}{0.4\pi} = \underline{\underline{5x10^5}}$ m/s

(b) $\lambda = 2\pi/\beta = \frac{2\pi}{0.4\pi} = \underline{\underline{5}}$ m

(c) $\delta = 1/\alpha = \frac{1}{0.4\pi} = \underline{\underline{0.796}}$ m

(d) $\eta = |\eta|\angle\theta_\eta, \theta_\eta = 45^o$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} \cong \sqrt{\frac{\mu}{\varepsilon}\frac{\omega\varepsilon}{\sigma}} = \sqrt{\frac{4\pi x10^{-7}x2\pi x10^8}{4}} = 14.05$$

$\eta = \underline{\underline{14.05\angle 45^o \quad \Omega}}$

Prob. 10.8 (a)

$$T = 1/f = 2\pi/\omega = \frac{2\pi}{\pi x10^8} = \underline{\underline{20 \text{ ns}}}$$

(b) Let $x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2}$$

But $\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}}\sqrt{x-1}$

$$\sqrt{x-1} = \frac{\alpha c}{\omega\sqrt{\frac{\mu_r\varepsilon_r}{2}}} = \frac{0.1x3x10^8}{\pi x10^8\sqrt{2}} = 0.06752 \longrightarrow x = 1.0046$$

$$\beta = \left(\frac{x+1}{x-1}\right)^{1/2}\alpha = \left(\frac{2.0046}{0.0046}\right)^{1/2} 0.1 = 2.088$$

$$\lambda = 2\pi/\beta = \frac{2\pi}{2.088} = \underline{\underline{3}}\text{ m}$$

(c) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x}} = \frac{377}{2\sqrt{1.0046}} = 188.1$

$$x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} = 1.0046$$

$$\frac{\sigma}{\omega\varepsilon} = 0.096 = \tan 2\theta_\eta \longrightarrow \theta_\eta = 2.74^o$$

$$\eta = 188.1\angle 2.74^o \quad \Omega$$

$$E_o = \eta H_o = 12x188.1 = 2256.84$$

$$a_E x a_H = a_k \longrightarrow a_E x a_x = a_y \longrightarrow a_E = a_z$$

$$E = \underline{\underline{2.256e^{-0.1y}\sin(\pi x10^8 t - 2.088y + 2.74^o)a_z}} \text{ kV/m}$$

(e) The phase difference is $\underline{\underline{2.74^o}}$.

Prob. 10.9 (a) $\gamma = \alpha + j\beta = \underline{\underline{0.05 + j2}}$ /m

(b) $\lambda = 2\pi/\beta = \pi = \underline{\underline{3.142}}$ m

(c) $u = \omega/\beta = \frac{2x10}{2} = \underline{\underline{10^8}}$ m/s

(d) $\delta = 1/\alpha = \dfrac{1}{0.05} = \underline{\underline{20}}$ m

Prob. 10.10 (a) $\beta = \omega / c = \dfrac{2\pi x 10^6}{3x10^8} = \underline{\underline{0.02094}}$ rad/m,

$$\lambda = 2\pi / \beta = \underline{\underline{300}} \text{ m}$$

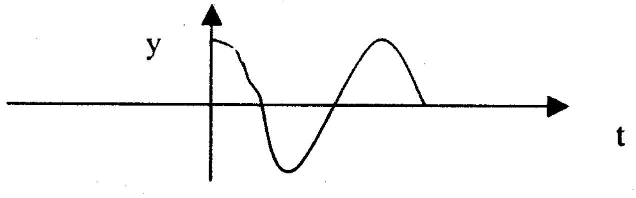
(b) When $z = 0$, $E_y = 10\cos\omega t$

$$z = \lambda / 4, \quad E_y = 10\cos(\omega t - \frac{2\pi}{\lambda}\frac{\lambda}{4}) = 10\sin\omega t$$

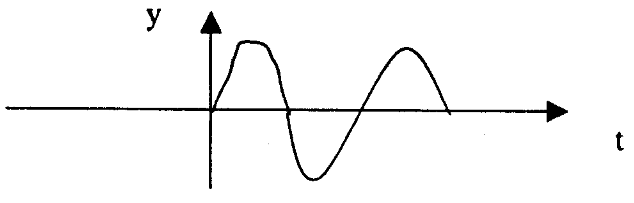
$$z = \lambda / 2, \quad E_y = 10\cos(\omega t - \pi) = -10\cos\omega t$$

Thus E is sketched below.

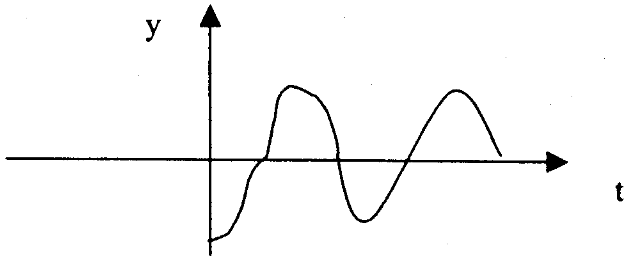
z = 0



$z = \lambda / 4$



$z = \lambda / 2$



(c)

$$H = \frac{1}{120\pi}\cos(2\pi x 10^6 t - 2\pi z / 300)a_x = \underline{\underline{26.53\cos(2\pi x 10^6 t - 0.02094)a_x}} \text{ A/m}$$

Prob. 10.11 (a) Along <u>-x direction.</u>

(b) $\beta = 6, \quad \omega = 2x10^8$,

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \beta c/\omega = \frac{6x3x10^8}{2x10^8} = 9 \longrightarrow \varepsilon_r = 81$$

$$\varepsilon = \varepsilon_o\varepsilon_r = \frac{10^{-9}}{36\pi}x81 = \underline{\underline{7.162x10^{-10}}} \text{ F/m}$$

(c) $\eta = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{\mu_r/\varepsilon_r} = \dfrac{120\pi}{9}$

$$E_o = H_o\eta = 25x10^{-3}x377/9 = 1.047$$

$$a_E x a_H = a_k \longrightarrow a_E x a_y = -a_x \longrightarrow a_E = a_z$$

$$E = \underline{\underline{1.047\sin(2x10^8 t + 6x)a_z}} \text{ V/m}$$

Prob. 10.12 $\beta = 4 \longrightarrow \lambda = 2\pi/\beta = \underline{\underline{1.571}}$ m

Also, $\beta = \omega/u = \omega\sqrt{\mu\varepsilon} = \dfrac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$

$$\omega = \frac{\beta c}{\sqrt{\mu_r\varepsilon_r}} = \frac{4x3x10^8}{\sqrt{4}} = \underline{\underline{6x10^8}} \text{ rad/s}$$

$$J_d = \nabla x H = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(z) & 0 & 0 \end{vmatrix} = \frac{\partial H_x}{\partial z}a_y$$

$$J_d = -40\cos(\omega t - 4z)x10^{-3}a_y = \underline{\underline{-40\cos(\omega t - 4z)a_y}} \text{ mA/m}^2$$

Prob. 10.13 (a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-6}}{2\pi x10^7 x5x\dfrac{10^{-9}}{36\pi}} = 3.6x10^{-4} << 1$

Thus, the material is lossless at this frequency.

(b) $\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi x10^7}{3x10^8}\sqrt{5x750} = \underline{\underline{12.83}}$ rad/m

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12.83} = \underline{\underline{0.49}}$ m

(c) Phase difference = βl = $\underline{\underline{25.66 \text{ rad}}}$

(d) $\eta = \sqrt{\mu/\varepsilon} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{750}{5}} = \underline{\underline{4617\Omega}}$

Prob. 10.14 If **A** is a uniform vector and $\Phi(r)$ is a scalar,

$\nabla x(\Phi A) = \nabla \Phi x A + \Phi(\nabla x A) = \nabla \Phi x A$

since $\nabla X A = 0$.

$$\nabla x E = (\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z) x E_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x a_x + k_y a_y + k_z a_z)e^{j(\quad)} x E_o$$

$$= jkxE_o e^{j(\quad)} = jkxE$$

Also, $-\frac{\partial B}{\partial t} = j\omega\mu H$. Hence $\nabla x E = -\frac{\partial B}{\partial t}$ becomes $kxE = \omega\mu H$

From this, $\underline{\underline{a_k x a_E = a_H}}$

Prob. 10.15

$$\nabla \bullet E = (\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z) \bullet E_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x a_x + k_y a_y + k_z a_z)e^{j(\quad)} \bullet E_o$$

$$= jk \bullet E_o e^{j(\quad)} = jk \bullet E = 0 \longrightarrow k \bullet E = 0$$

Similarly,

$$\nabla \bullet H = jk \bullet H = 0 \longrightarrow k \bullet H = 0$$

It has been shown in Prob. 10.14 that

$$\nabla x E = -\frac{\partial B}{\partial t} \longrightarrow kxE = \omega\mu H$$

Similarly,

$$\nabla x H = \frac{\partial D}{\partial t} \longrightarrow kx H = -\varepsilon \omega E$$

From $kxE = \omega \mu H$, $a_k x a_E = a_H$ and

From $kxH = -\varepsilon\omega E$, $a_k x a_H = -a_E$

Prob. 10.16 (a)

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_r} \longrightarrow \sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{5x3x10^8}{2\pi x10^8} = \frac{15}{2\pi}$$

$$\underline{\underline{\varepsilon_r = 5.6993}}$$

(b) $\lambda = 2\pi / \beta = 2\pi / 5 = \underline{\underline{1.2566}}$ m

$$u = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3x10^8}{\frac{15}{2\pi}} = \underline{\underline{1.257x10^8}} \text{ m/s}$$

(c) $\eta = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{120\pi}{\frac{15}{2\pi}} = \underline{\underline{157.91\Omega}}$

(d) $a_E x a_H = a_k \longrightarrow a_E x a_z = a_x \longrightarrow a_E = \underline{\underline{a_y}}$

(e) $E = 30x10^{-3}(157.91)\sin(\omega t - \beta x)a_E = \underline{\underline{4.737\sin(2\pi x10^8 t - 5x)a_y}}$ V/m

(f) $J_d = \frac{\partial D}{\partial t} = \nabla x H = \underline{\underline{0.15\cos(2\pi x10^8 t - 5x)a_y}}$ A/m

Prob. 10.17 $\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\varepsilon_r}\sqrt{\mu_r}$, $\mu_r = 1$

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{8x3x10^8}{10^9} = 2.4 \longrightarrow \underline{\underline{\varepsilon_r = 5.76}}$$

Let $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$.

$$E_1 = 50\cos(10^9 t - 8x)a_y, \qquad E_2 = 40\sin(10^9 t - 8x)a_z$$

$$H_1 = H_{o1}\cos(10^9 t - 8x)a_{H1}, \qquad H_{o1} = \frac{50x2.4}{120\pi} = \frac{1}{\pi}$$

$$a_{E1}xa_{H1} = a_{k1} \longrightarrow a_y xa_{H1} = a_x \longrightarrow a_{H1} = a_z$$

$$H_1 = \frac{1}{\pi}\cos(10^9 t - 8x)a_z,$$

$$H_2 = H_{o2}\sin(10^9 t - 8x)a_{H2}, \qquad H_{o2} = \frac{40x2.4}{120\pi} = \frac{0.8}{\pi}$$

$$a_{E2}xa_{H2} = a_{k2} \longrightarrow a_z xa_{H2} = a_x \longrightarrow a_{H2} = -a_y$$

$$H_2 = -\frac{0.8}{\pi}\sin(10^9 t - 8x)a_y,$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = \underline{\underline{-0.2546\sin(10^9 t - 8x)a_y + 0.3183\cos(10^9 t - 8x)a_z}} \quad \text{A/m}$$

Prob. 10.18 $\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi x10^7}{3x10^8}(10) = \underline{\underline{2.0943}}$ rad/m

$$H = -\frac{1}{\mu}\int \nabla xEdt$$

$$\nabla xE = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x) & E_z(x) \end{vmatrix} = -\frac{\partial E_z}{\partial x}a_y + \frac{\partial E_y}{\partial x}a_z = -10\beta\sin(\omega t - \beta x)(a_y - a_z)$$

$$H = -\frac{10\beta}{\omega\mu}\cos(\omega t - \beta x)(a_y - a_z) = -\frac{10x2\pi/3}{2\pi x10^7 x50x4\pi x10^{-7}}\cos(\omega t - \beta x)(a_y - a_z)$$

$$H = \underline{\underline{5.305\cos(2\pi x10^7 t - 2.0943x)(-a_y + a_z)}} \quad \text{mA/m}$$

Prob. 10.19 For a good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$, say $\frac{\sigma}{\omega\varepsilon} > 100$

(a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-2}}{2\pi x 8x10^6 x 15 x \dfrac{10^{-9}}{36\pi}} = 1.5 \longrightarrow$ lossy

No, not conducting.

(b) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{0.025}{2\pi x 8x10^6 x 16 x \dfrac{10^{-9}}{36\pi}} = 3.515 \longrightarrow$ lossy

No, not conducting.

(c) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{25}{2\pi x 8x10^6 x 81 x \dfrac{10^{-9}}{36\pi}} = 694.4 \longrightarrow$ conducting

Yes, conducting.

Prob. 10.20

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi f}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1.0049} - 1\right]} = \frac{2\pi x 6x10^6}{3x10^8}\sqrt{\frac{4}{2}x 2.447x10^{-3}}$$

$\alpha = 8.791x10^{-3}$

$\delta = 1/\alpha = 113.75$ m

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} = \frac{4\pi}{100}\sqrt{\frac{4}{2}\left[\sqrt{1.0049} + 1\right]} = 0.2515$$

$$u = \omega / \beta = \frac{2\pi x 6x10^6}{0.2525} = \underline{\underline{1.5x10^8}} \text{ m/s}$$

Prob. 10.21 $\quad 0.4E_o = E_o e^{-\alpha z} \longrightarrow \dfrac{1}{0.4} = e^{2\alpha}$

Or $\quad \alpha = \dfrac{1}{2}\ln\dfrac{1}{0.4} = 0.4581 \longrightarrow \delta = 1/\alpha = \underline{\underline{2.183}}$ m

$\lambda = 2\pi / \beta = 2\pi / 1.6$

$$u = f\lambda = 10^{-} x \frac{2\pi}{1.6} = \underline{\underline{3.927x10^{-}}} \text{ m/s}$$

Prob. 10.22 (a)

$$R_{dc} = \frac{l}{\sigma S} = \frac{l}{\sigma \pi a^2} = \frac{600}{5.8x10^7 x\pi x(1.2)^2 x10^{-6}} = 2.287\Omega$$

(b) $R_{ac} = \dfrac{l}{\sigma 2\pi a\delta}$. At 100 MHz, $\delta = 6.6x10^{-3}$ mm for copper (see Table 10.2).

$$R_{ac} = \frac{600}{5.8x10^7 x2\pi x(1.2)x6.6x10^{-3} x10^{-6}} = \underline{\underline{207.88\Omega}}$$

(c) $\dfrac{R_{ac}}{R_{dc}} = \dfrac{a}{2\delta} = 1 \longrightarrow \delta = a/2 = \dfrac{66.1x10^{-3}}{\sqrt{f}}$

$$\sqrt{f} = \frac{66.1x2x10^{-3}}{a} = \frac{66.1x2}{1.2} \longrightarrow f = \underline{\underline{12.127}} \text{ kHz}$$

Prob. 10.23

$$\omega = 10^6\pi = 2\pi f \longrightarrow f = 0.5x10^6$$

$$\delta = \frac{1}{\sqrt{\pi f\sigma\mu}} = \frac{1}{\sqrt{\pi x0.5x10^6 x3.5x10^7 x4\pi x10^{-7}}} = \underline{\underline{0.1203}} \text{ mm}$$

$$R_{ac} = \frac{l}{\sigma\delta w}$$

since δ is very small, $w \simeq 2\pi\rho_{outer}$

$$R_{ac} = \frac{l}{\sigma 2\pi\rho_{outer}\delta} = \frac{40}{3.5x10^7 x0.1203x2\pi x12x10^{-6}} = \underline{\underline{0.126\Omega}}$$

Prob. 10.24 $\alpha = \beta = 1/\delta$

$$\lambda = 2\pi/\beta = 2\pi\delta = 6.283\delta \longrightarrow \delta = 0.1591\lambda$$

showing that δ is shorter than λ.

Prob. 10.25

$$t = 5\delta = \frac{5}{\sqrt{\pi f\mu\sigma}} = \frac{5}{\sqrt{\pi x12x10^9 x4\pi x10^{-7} x6.1x10^7}} = \underline{\underline{2.94x10^{-6}}} \text{ m}$$

Prob. 10.26 (a)

$$E = \text{Re}[E_s e^{j\omega t}] = (5a_x + 12a_y)e^{-0.2z}\cos(\omega t - 3.4z)$$

At z = 4m, t = T/8, $\omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$$E = (5a_x + 12a_y)e^{-0.8}\cos(\pi/4 - 13.6)$$

$$|E| = 13e^{-0.8}|\cos(\pi/4 - 13.6)| = \underline{\underline{5.662}}$$

(b) loss $= \alpha\Delta z = 0.2(3) = 0.6$ Np. Since 1 Np = 8.686 dB,

loss = 0.6 x 8.686 = $\underline{\underline{5.212 \text{ dB}}}$

(c) Let $x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2/3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega\sqrt{\mu\varepsilon/2}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r/2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2x3x10^3}{10^8\sqrt{0.00694}} = 2.4 \quad \longrightarrow \quad \varepsilon_r = 11.52$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o}}\cdot\frac{1}{\sqrt{\varepsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{11.52x1.00694}} = 32.5$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2 - 1} = 0.118 \quad \longrightarrow \quad \theta_\eta = 3.365^o$$

$$\eta = 32.5\angle 3.365^o$$

$$H_s = a_k x \frac{E_s}{\eta} = \frac{a_z}{\eta} x(5a_x + 12a_y)e^{-\gamma z} = \frac{(5a_x + 12a_y)}{|\eta|} e^{-j3.365^o} e^{-\gamma z}$$

$$H = (-369.2a_x + 153.8a_y)e^{-0.2z} \cos(\omega t - 3.4z - 3.365^o) \quad \text{mA}$$

$$P = ExH = \begin{vmatrix} 5 & 12 & 0 \\ -369.2 & 153.8 & 0 \end{vmatrix} x10^{-3} e^{-0.4z} \cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)$$

$$P = 5.2e^{-0.4z} \cos(\omega t - 3.4z)\cos(\omega t - 3.4z - 3.365^o)a_z$$

At z = 4, t = T/4,

$$P = 5.2e^{-1.6} \cos(\pi / 4 - 13.6)\cos(\pi / 4 - 13.6 - 0.0587)a_z = \underline{\underline{0.9702a_z \text{W/m}^2}}$$

Prob. 10.27 (a) This is a lossl--s medium,

$$\beta = \omega\sqrt{\mu\varepsilon}, \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta = \frac{\omega\mu_o}{\beta} = \frac{2\pi x10^8 x4\pi x10^{-7}}{6} = \underline{\underline{131.6\Omega}}$$

(b) $E_o = \eta H_o = 131.6x30x10^{-3} = 3.948$

$$a_E x a_H = a_k \longrightarrow a_E x a_y = a_x \longrightarrow a_E = -a_z$$

$$P = ExH = \eta H_o^2 \cos^2(2\pi x10^8 t - 6x)a_x = \underline{\underline{0.1184\cos^2(2\pi x10^8 t - 6x)a_x \ \text{W/m}^2}}$$

(c) $\mathcal{P}_{ave} = \frac{1}{2}\eta H_o^2 = 0.0592a_x \ \text{W/m}^2$

$$P_{ave} = \int \mathcal{P}_{ave} \bullet dS = \mathcal{P}_{ave} \bullet S = 0.0592x3x2 = \underline{\underline{0.3535 \text{ W}}}$$

Prob. 10.28 Let $E_s = E_r + jE_i$ and $H_s = H_r + jH_i$

$$E = \text{Re}(E_s e^{j\omega t}) = E_r \cos\omega t - E_i \sin\omega t$$

Similarly,

$$H = H_r \cos\omega t - H_i \sin\omega t$$

$$\mathcal{P} = ExH = E_r x H_r \cos^2 \omega t + E_i x H_i \sin^2 \omega t - \frac{1}{2}(E_r x H_i + E_i x H_r) \sin\ 2\omega t$$

$$\mathcal{P}_{ave} = \frac{1}{T}\int_0^T \mathcal{P} dt = \frac{1}{T}\int_0^T \cos^2 \omega\, dt (E_r x H_r) + \frac{1}{T}\int_0^T \sin^2 \omega\, dt (E_i x H_i) - \frac{1}{2T}\int_0^T \sin 2\omega\, dt (E_i x H_i + F_i x H_r)$$

$$= \frac{1}{2}(E_r x H_r + E_i x H_i) = \frac{1}{2}\mathrm{Re}[(E_r + jE_i)x(H_r - jH_i)]$$

$$\mathcal{P}_{ave} = \frac{1}{2}\ \mathrm{Re}(E_s x H_s^*)$$

as required.

Prob. 10.29 (a)

$$u = \omega / \beta \quad \longrightarrow \quad \omega = u\beta = \frac{\beta}{c}\frac{1}{\sqrt{4.5}} = \frac{2x3x10^8}{\sqrt{4.5}} = \underline{\underline{2.828x10^8}}\ \text{rad/s}$$

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7\Omega$$

$$H = a_k x \frac{E}{\eta} = \frac{a_z}{\eta} x \frac{40}{\rho}\sin(\omega t - 2z)a_\rho = \underline{\underline{\frac{0.225}{\rho}\sin(\omega t - 2z)a_\phi}}\ \ \text{A/m}$$

(b) $$\mathcal{P} = ExH = \underline{\underline{\frac{9}{\rho^2}\sin^2(\omega t - 2z)a_z\ \text{W/m}^2}}$$

(c) $$\mathcal{P}_{ave} = \frac{4.5}{\rho^2}a_z,\ \mathrm{dS} = \rho\, d\phi\, d\rho\, a_z$$

$$P_{ave} = \int \mathcal{P}_{ave} \bullet \mathrm{dS} = 4.6 \int_{2mm}^{3mm} \frac{d\rho}{\rho}\int_0^{2\pi} d\phi = 4.5\ln(3/2)(2\pi) = \underline{\underline{11.46\ \text{W}}}$$

Prob. 10.30 (a) $P_{i,ave} = \dfrac{E_{io}^2}{2\eta_1}$, $\quad P_{r,ave} = \dfrac{E_{ro}^2}{2\eta_1}$, $\quad P_{t,ave} = \dfrac{E_{to}^2}{2\eta_2}$

$$R = \frac{P_{r,ave}}{P_{i,ave}} = \frac{E_{ro}^2}{E_{io}^2} = \Gamma^2 = \underline{\underline{\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)^2}}$$

$$R = \left(\frac{\sqrt{\frac{\mu_o}{\varepsilon_2}} - \sqrt{\frac{\mu_o}{\varepsilon_1}}}{\sqrt{\frac{\mu_o}{\varepsilon_2}} + \sqrt{\frac{\mu_o}{\varepsilon_1}}} \right)^2 = \left(\frac{\sqrt{\mu_o \varepsilon_1} - \sqrt{\mu_o \varepsilon_2}}{\sqrt{\mu_o \varepsilon_1} + \sqrt{\mu_o \varepsilon_2}} \right)^2$$

Since $n_1 = c\sqrt{\mu_1 \varepsilon_1} = c\sqrt{\mu_o \varepsilon_1}$, $\quad n_2 = c\sqrt{\mu_o \varepsilon_2}$,

$$R = \left(\frac{n_1 + n_2}{n_1 + n_2} \right)^2$$

$$T = \frac{P_{t,ave}}{P_{i,ave}} = \frac{\eta_1}{\eta_2} \frac{E_{to}^2}{E_{io}^2} = \frac{\eta_1}{\eta_2} \tau^2 = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

(b) If $P_{r,ave} = P_{t,ave} \longrightarrow RP_{i,ave} = TP_{i,ave} \longrightarrow R = T$

i.e. $(n_1 - n_2)^2 = 4 n_1 n_2 \quad \longrightarrow \quad n_1^2 - 6 n_1 n_2 + n_2^2 = 0$

$$\frac{n_1}{n_2} = 3 \pm \sqrt{8} = \underline{\underline{5.828 \quad \text{or} \quad 0.1716}}$$

Prob. 10.31 (a) $\eta_1 = \eta_o$, $\quad \eta_o = \sqrt{\frac{\mu}{\varepsilon}} = \eta_o / 2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o/2 - \eta_o}{3\eta_o/2} = \underline{\underline{-1/3}}, \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{\eta_o}{3\eta_o/2} = \underline{\underline{2/3}}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \underline{\underline{2}}$$

(b) $\quad E_{or} = \Gamma E_{oi} = -\frac{1}{3} x(30) = -10$

$$\underline{\underline{E_r = -10\cos(\omega t + z)a_x}} \text{ V/m}$$

Let $H_r = H_{or}\cos(\omega t + z)a_H$

$$a_E x a_H = a_k \longrightarrow -a_k x a_H = -a_z \longrightarrow a_H = a_x$$

$$H_r = \frac{10}{120\pi}\cos(\omega t + z)a_y = \underline{\underline{26.53\cos(\omega t + z)a_y}} \text{ mA/m}$$

Prob. 10.32 (a) $\eta_1 = \eta_o$

$$E_i = E_{io}\sin(\omega t - 5x)a_E$$

$$E_{io} = H_{io}\eta_o = 120\pi x 4 = 480\pi$$

$$a_E x a_H = a_k \longrightarrow a_E x a_y = a_x \longrightarrow a_E = -a_z$$

$$E_i = -480\pi\sin(\omega t - 5x)a_z$$

$$\eta_2 = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -1/3, \qquad \tau = 1 + \Gamma = 2/3$$

$$E_{ro} = \Gamma E_{io} = (-1/3)(480\pi) = -160\pi$$

$$E_r = 160\pi\sin(\omega t + 5x)a_z$$

$$E_1 = E_i + E_r = \underline{\underline{-1.508\sin(\omega t - 5x)a_z + 0.503\sin(\omega t + 5x)a_z}} \text{ kV/m}$$

(b) $E_{to} = \tau E_{io} = (2/3)(480\pi) = 320\pi$

$$\mathcal{P} = \frac{E_{to}^2}{2\eta_2}a_x = \frac{(320\pi)^2}{2(60\pi)}a_x = \underline{\underline{2.68a_x \text{ kW/m}^2}}$$

(c) $s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \underline{\underline{2}}$

Prob. 10.33 $\eta_1 = \eta_o = 120\pi$, $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad (1)$$

But $E_{ro} = \eta_o H_{ro}$ (2)

Combining (1) and (2),

$$E_{ro} = \eta_o H_{ro} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{io} \quad \longrightarrow \quad \eta_o = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \frac{E_{io}}{H_{ro}}$$

But $\frac{E_{io}}{H_{ro}} = \frac{3.6}{1.2x10^{-3}} = 3000$

$$\eta_o = 3000\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \quad \longrightarrow \quad 377 = 3000\left(\frac{\eta_2 - 377}{\eta_2 + 377}\right)$$

Thus, $\eta_2 = 485.37$. Since $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$,

$$\mu_2 = \varepsilon_o \varepsilon_r \eta_2^2 = \frac{10^{-9}}{36\pi} x 12.5 x (485.37)^2 = \underline{\underline{2.604x10^{-5}}} \text{ H/m}$$

Prob. 10.34 $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \eta_o / 2, \quad \eta_2 = \eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1/3, \qquad \tau = 1 + \Gamma = 4/3$$

$$E_{or} = \Gamma E_{io} = (1/3)(5) = 5/4, \qquad E_{ot} = \tau E_{io} = 20/3$$

$$\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{10^8}{3x10^8}\sqrt{4} = 2/3$$

(a) $E_r = \frac{5}{3}\cos(10^8 t - 2y/3) a_z$

$$E_1 = E_i + E_r = \underline{\underline{5\sin(10^8 t + \frac{2}{3}y) a_z + \frac{5}{3}\cos(10^8 t - \frac{2}{3}y) a_z}} \text{ V/m}$$

(b) $\mathcal{P}_{ave1} = \frac{E_{io}^2}{2\eta_1}(-a_y) + \frac{E_{ro}^2}{2\eta_1}(+a_y) = \frac{25}{2(60\pi)}(1 - \frac{1}{9})(-a_y) = \underline{\underline{-0.0589 a_y \text{ W/m}^2}}$

(c) $\mathcal{P}_{ave2} = \frac{E_{to}^2}{2\eta_2}(-a_y) = \frac{400}{9(2)(120\pi)}(-a_y) = \underline{\underline{-0.0589 a_y \text{ W/m}^2}}$

Prob. 10.35 (a) $\beta = 1 = \omega / u = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$

$$\omega = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3x10^8}{\sqrt{3x12}} = \underline{\underline{0.5x10^8 \text{ rad/s}}}$$

(b) $\eta_1 = \eta_o, \qquad \eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_o\sqrt{\frac{3}{12}} = \eta_o / 2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3, \qquad \tau = 1 + \Gamma = 2/3$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \underline{\underline{2}}$$

(c) Let $H_r = H_{or}\cos(\omega t + z)a_H$, where

$$E_r = -\frac{1}{3}(3)\cos(\omega t + z)a_y = -10\cos(\omega t + z)a_y, \qquad H_{or} = \frac{10}{\eta_o} = \frac{10}{120\pi}$$

$$a_E x a_H = a_k \longrightarrow -a_y x a_H = -a_z \longrightarrow a_H = -a_x$$

$$H_r = -\frac{10}{120\pi}\cos(0.5x10^8 t + z)a_x \text{ A/m} = -26.53\cos(0.5x10^8 t + z)a_x \text{ mA/m}$$

Prob. 10.36 (a)

$$a_E x a_H = a_k \longrightarrow a_E x a_z = a_x \longrightarrow a_E = -a_y$$

i.e. polarization is <u>along the y-axis</u>.

(b) $\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\pi x 30x10^6}{3x10^8}\sqrt{4x9} = \underline{\underline{3.77}}$ rad/m

(c) $J_d = \nabla x H = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ 0 & 0 & H_z(x,t) \end{vmatrix} = -\frac{\partial H_z}{\partial x}a_y$

$$= -10\beta\cos(\omega t + \beta x)a_y = \underline{\underline{-37.6\cos(\omega t + \beta x)a_y \text{ mA/m}}}$$

(d) $\eta_2 = \eta_o, \qquad \eta_1 = \eta_o\sqrt{\frac{4}{9}} = \frac{2}{3}\eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1/5, \qquad \tau = 1 + \Gamma = 6/5$$

$$E_i = 10\eta_1 \sin(\omega t + \beta x) a_E \text{ mV/m}, \quad a_E = -a_y$$

$$E_r = \Gamma 10\eta_1 \sin(\omega t - \beta x)(-a_y) \text{ mV/m}$$

$$a_E x a_H = a_k \longrightarrow -a_y x a_H = a_x \longrightarrow a_H = -a_z$$

$$H_r = \Gamma 10 \sin(\omega t - \beta x)(-a_z) \text{ mA/m} = \underline{\underline{-2\sin(\omega t - \beta x) a_z \text{ mA/m}}}$$

$$E_t = \tau 10\eta_1 \sin(\omega t + \beta x)(-a_y) \text{ mV/m}$$

$$a_E x a_H = a_k \longrightarrow -a_y x a_H = -a_x \longrightarrow a_H = a_z$$

$$H_t = 10(6/5)(\eta_1/\eta_2)\sin(\omega t + \beta x) a_z \text{ mA/m} = \underline{\underline{8\sin(\omega t + \beta x) a_z \text{ mA/m}}}$$

(e) $$\mathcal{P}_{\text{ave1}} = \frac{E_{io}^2}{2\eta_1}(-a_x) + \frac{E_{ro}^2}{2\eta_1}(+a_x) = \frac{-E_{io}^2}{2\eta_1}(1 - \Gamma^2) a_x$$

$$= -\frac{\eta_1^2 H_{io}^2}{2\eta_1}(1 - \Gamma^2) a_x = -\frac{1}{3}\eta_o 100(1 - \frac{1}{25}) a_x = \underline{\underline{-0.012064 a_x \text{ W/m}^2}}$$

$$E_{ot} = \tau E_{oi} = \tau \eta_1 H_{io}$$

$$\mathcal{P}_{\text{ave2}} = \frac{E_{to}^2}{2\eta_2}(-a_x) = \frac{\tau^2 \eta_1^2 H_{io}^2}{2\eta_2}(-a_x) = 32\eta_o(-a_x)\ \mu\text{W/m}^2 = \underline{\underline{-0.012064 a_x \text{ W/m}^2}}$$

Prob. 10.37 (a) In air, $\beta_1 = 1, \lambda_1 = 2\pi/\beta_1 = 2\pi = \underline{\underline{6.283 \text{ m}}}$

$$\omega = \beta_1 c = \underline{\underline{3x10^8 \text{ rad/s}}}$$

In the dielectric medium, ω is the same .

$$\omega = \underline{\underline{3x10^8 \text{ rad/s}}}$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r2}} = \beta_1\sqrt{\varepsilon_{r2}} = \sqrt{3}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\sqrt{3}} = \underline{\underline{3.6276\text{ m}}}$$

(b) $$H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = 0.0265$$

$$a_H = a_k x a_E = a_z x a_y = a_x$$

$$H_t = \underline{\underline{-26.5\cos(\omega t - z)a_x \text{ mA/m}}}$$

(c) $\eta_1 = \eta_o, \quad \eta_1 = \eta_o/\sqrt{3}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1/\sqrt{3}) - 1}{(1/\sqrt{3}) + 1} = \underline{\underline{-0.268}}, \qquad \tau = 1 + \Gamma = \underline{\underline{0.732}}$$

(d) $E_{to} = \tau E_{io} = 7.32, \quad E_{ro} = \Gamma E_{io} = -2.68$

$$E_1 = E_i + E_r = \underline{\underline{10\cos(\omega t - z)a_y - 2.68\cos(\omega t + z)a_y \text{ V/m}}}$$

$$E_2 = E_t = \underline{\underline{7.32\cos(\omega t - z)a_y \text{ V/m}}}$$

$$\mathcal{P}_{ave1} = \frac{1}{2\eta_1}(a_z)[E_{io}^2 - E_{ro}^2] = \frac{1}{2(120\pi)}(a_z)(10^2 - 2.68^2) = \underline{\underline{0.1231a_z \text{ W/m}^2}}$$

$$\mathcal{P}_{ave2} = \frac{E_{to}^2}{2\eta_2}(a_z) = \frac{\sqrt{3}}{2x120\pi}(7.32)^2(a_z) = \underline{\underline{0.1231a_z \text{ W/m}^2}}$$

Prob. 10.38 (a) $\omega = \beta c = 3x3x10^8 = \underline{\underline{9x10^8 \text{ rad/s}}}$

(b) $\lambda = 2\pi/\beta = 2\pi/3 = \underline{\underline{2.094}}$

(c) $$\frac{\sigma}{\omega\varepsilon} = \frac{4}{9x10^8 x 80x10^{-9}/36\pi} = 2\pi = \underline{\underline{6.288}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 6.288 \longrightarrow \theta_\eta = 40.47^o$$

$$|\eta_2| = \frac{\sqrt{\mu_2/\varepsilon_2}}{\sqrt[4]{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{377/\sqrt{80}}{\sqrt[4]{1+4\pi^2}} = 16.71$$

$$\eta_2 = \underline{\underline{16.71\angle 40.47^o\ \Omega}}$$

(d) $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{16.71\angle 40.47^o - 377}{16.71\angle 40.47^o + 377} = 0.935\angle 176.7^o$$

$$E_{or} = \Gamma E_{oi} = 9.35\angle 176.7^o$$

$$E_r = \underline{\underline{9.35\sin(\omega t - 3z + 176.7)a_x \text{ V/m}}}$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{\mu_{r2}\varepsilon_{r2}}{2}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2} - 1\right]} = \frac{9x10^9}{3x10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} - 1\right]} = 43.94 \text{ Np/m}$$

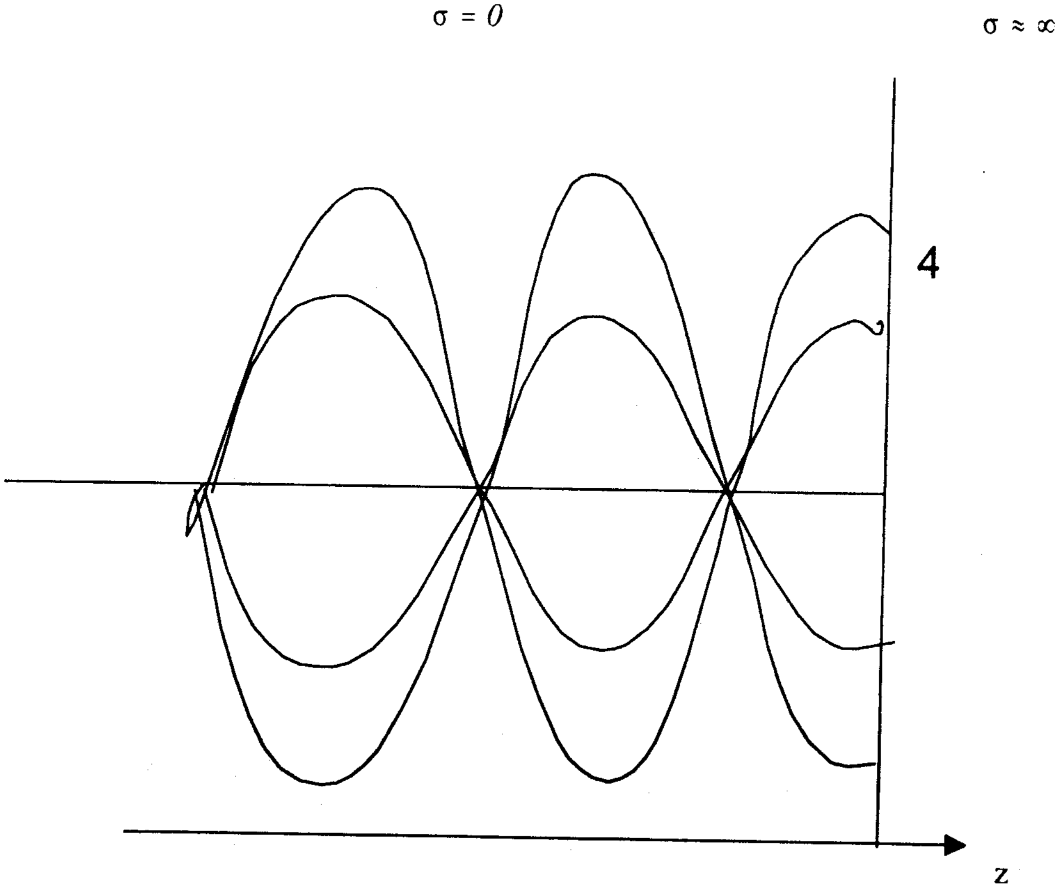
$$\beta_2 = \frac{9x10^9}{3x10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} + 1\right]} = 51.48 \text{ rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2x16.71\angle 40.47^o}{16.71\angle 40.47^o + 377} = 0.0857\angle 38.89^o$$

$$E_{ot} = \tau E_o = 0.857\angle 38.59^o$$

$$\underline{\underline{E_t = 0.857e^{43.94z}\sin(9x10^8 t + 51.48z + 38.89^o) \text{ V/m}}}$$

Prob. 10.39



Curve 0 is at t = 0; curve 1 is at t = T/8; curve 2 is at t = T/4; curve 3 is at t = 3T/8, etc.

Prob. 10.40 Since $\mu_o = \mu_1 = \mu_2$,

$$\sin\theta_{t1} = \sin\theta_i \sqrt{\frac{\varepsilon_o}{\varepsilon_1}} = \frac{\sin 45^o}{\sqrt{4.5}} = 0.3333 \longrightarrow \underline{\underline{\theta_{t1} = 19.47^o}}$$

$$\sin\theta_{t2} = \sin\theta_{t1} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{1}{3}\sqrt{\frac{2.25}{4.5}} = 0.2357 \longrightarrow \underline{\underline{\theta_{t2} = 13.63^o}}$$

Prob. 10.41

$$E_s = \frac{20(e^{jk_x x} - e^{-jk_x x})}{2} \frac{(e^{jk_y y} - e^{-jk_y y})}{2} a_z$$

$$= -j5\left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)}\right]a_z$$

which consists of four plane waves.

$$\nabla x E_s = -j\omega\mu_o H_s \quad \longrightarrow \quad H_s = \frac{j}{\omega\mu_o}\nabla x E_s = \frac{j}{\omega\mu_o}\left(\frac{\partial E_z}{\partial y}a_x - \frac{\partial E_z}{\partial x}a_y\right)$$

$$H_s = -\frac{j20}{\omega\mu_o}\left[k_y \sin(k_x x)\sin(k_y y)a_x + k_x \cos(k_x x)\cos(k_y y)a_y\right]$$

Prob. 10.42 If $\mu_o = \mu_1 = \mu_2, \quad \eta_1 = \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}, \eta_2 = \frac{\eta_o}{\sqrt{\varepsilon_{r2}}}$

$$\Gamma_{\backslash\backslash} = \frac{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t - \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}$$

$$\sqrt{\varepsilon_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}}\sin\theta_t \quad \longrightarrow \quad \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} = \frac{\sin\theta_i}{\sin\theta_t}$$

$$\Gamma_{\backslash\backslash} = \frac{\cos\theta_t - \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}{\cos\theta_t + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i}$$

Dividing both numerator and denominator by $\cos\theta_i\cos\theta_t$ gives

$$\Gamma_{\backslash\backslash} = \frac{\tan\theta_t - \tan\theta_i}{\tan\theta_t + \tan\theta_i} = \frac{\frac{\tan\theta_t - \tan\theta_i}{1 + \tan\theta_t\tan\theta_i}}{\frac{\tan\theta_t + \tan\theta_i}{1 + \tan\theta_t\tan\theta_i}} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}$$

Similarly,

$$\tau_{\backslash} = \frac{\frac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i} = \frac{2\cos\theta_i}{\cos\theta_t + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}$$

$$= \frac{2\cos\theta_i\sin\theta_t}{\sin\theta_t\cos\theta_t(\sin^2\theta_i + \cos^2\theta_i) + \sin\theta_i\cos\theta_i(\sin^2\theta_t + \cos^2\theta_t)}$$

$$= \frac{2\cos\theta_i \sin\theta_t}{(\sin\theta_t \cos\theta_i + \sin\theta_i \cos\theta_t)(\cos\theta_i \cos\theta_t + \sin\theta_i \sin\theta_t)}$$

$$= \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$\Gamma_\perp = \frac{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{\cos\theta_i - \frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t}{\cos\theta_i + \frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\tau_\perp = \frac{\frac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)}$$

Prob. 10.43 (a) $k_i = 4a_y + 3a_z$

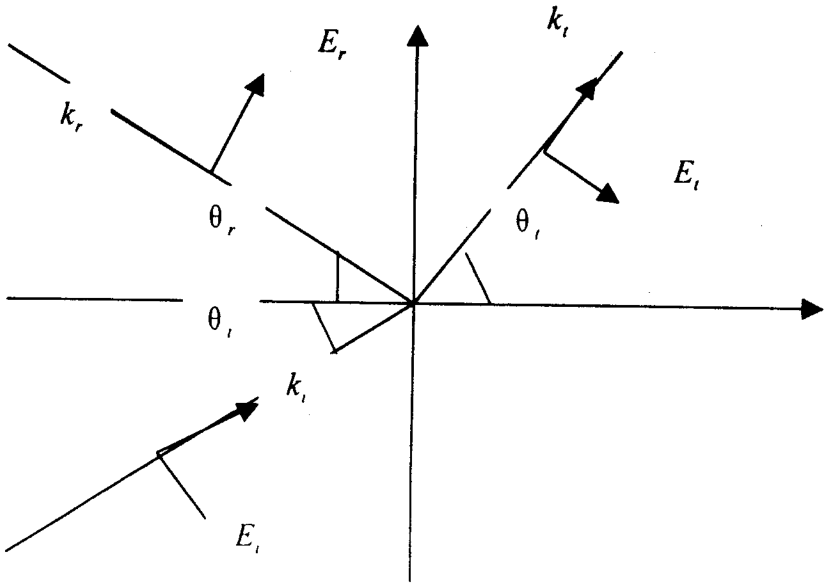
$k_i \bullet a_n = k_i \cos\theta_i \longrightarrow \cos\theta_i = 4/5 \longrightarrow \underline{\underline{\theta_i = 36.87^o}}$

(b)

$$\mathcal{P}_{ave} = \frac{1}{2}\,\mathrm{Re}(E_s x H_s^*) = \frac{E_o^2}{2\eta}a_k = \frac{(\sqrt{8^2+6^2})^2}{2x120\pi}\,\frac{(3a_y + 4a_z)}{5} = \underline{\underline{79.58a_y + 106.1a_z \text{ mW/m}^2}}$$

(c) $\theta_r = \theta_i = 36.87^o$. Let

$$E_r = (E_{ry}a_x + E_{rz}a_z)\sin(\omega t - k_r \bullet r)$$



From the figure, $k_r = k_{rz}a_z - k_{ry}a_y$. But $k_r = k_i = 5$

$$k_{rz} = k_r \sin\theta_r = 5(3/5) = 3, \qquad k_{ry} = k_r \cos\theta_r = 5(4/5) = 4,$$

Hence, $k_r = -4a_y + 3a_z$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{3/5}{\sqrt{4}} = 0.3$$

$$\theta_t = 17.46, \cos\theta_t = 0.9539, \qquad \eta_1 = \eta_o = 120\pi, \eta_2 = \eta_o/2 = 60\pi$$

$$\Gamma_{//} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\frac{\eta_o}{2}(0.9539) - \eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = -0.253$$

$$E_{ro} = \Gamma_{//}E_{io} = -0.253(10) = -2.53$$

But $(E_{ry}a_y + E_{rz}a_z) = E_{ro}(\sin\theta_r a_y + \cos\theta_r a_z) = -2.53(\frac{3}{5}a_y + \frac{4}{5}a_z)$

$$\underline{\underline{E_r = -(1.518a_y + 2.024a_z)\sin(\omega t + 4y - 3z)}} \text{ V/m}$$

Similarly, let

$$E_t = (E_{ty}a_y + E_{tz}a_z)\sin(\omega t - k_t \bullet r)$$

$$k_t = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{4\mu_o\varepsilon_o}$$

But $k_i = \beta_1 = \omega\sqrt{\mu_o\varepsilon_o}$

$$\frac{k_t}{k_i} = 2 \quad \longrightarrow \quad k_t = 2k_i = 10$$

$$k_{ty} = k_t\cos\theta_t = 9.539, \qquad k_{tz} = k_t\sin\theta_t = 3,$$

$$k_t = 9.539a_y + 3a_z$$

Note that $k_{iz} = k_{rz} = k_{tz} = 3$

$$\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{\eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = 0.6265$$

$$E_{to} = \tau_{\parallel} E_{io} = 0.265$$

But

$$(E_{ty} a_y + E_{tz} a_z) = E_{to}(\sin\theta_t a_y - \cos\theta_t a_z) = 0.256(0.3a_y - 0.9539a_z)$$

Hence,

$$\underline{\underline{E_t = (1.877a_y - 5.968a_z)\sin(\omega t - 9.539y - 3z) \text{ V/m}}}$$

Prob. 10.44 (a)

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} \quad \frac{1}{\sqrt{8}} \longrightarrow \underline{\underline{\theta_i = \theta_r = 19.47^o}}$$

$$\sin\theta_t = \sin\theta_i \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}(3) = 1 \longrightarrow \underline{\underline{\theta_t = 90^o}}$$

(b) $\beta_1 = \frac{\omega}{c}\sqrt{\varepsilon_{r1}} = \frac{10^9}{3x10^8} x3 = 10 = k\sqrt{1+8} = 3k \longrightarrow \underline{\underline{k = 3.333}}$

(c) $\lambda = 2\pi/\beta, \quad \lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283}} \text{ m}$

$\beta_2 = \omega/c = 10/3, \quad \lambda_2 = 2\pi/\beta_2 = 2\pi x3/10 = \underline{\underline{1.885}} \text{ m}$

(d) $E_i = \eta_1 a_k x H_i = 40\pi \frac{(a_x + \sqrt{8}a_z)}{3} x0.2\cos(\omega t - k \bullet r)a_y$

$$\underline{\underline{= (-213.3a_x + 75.4a_z)\cos(10^9 t - kx - k\sqrt{8}z) \text{ V/m}}}$$

(e) $\tau_{//} = \frac{2\cos\theta_t \sin\theta_i}{\sin(\theta_t + \theta_i)\cos(\theta_t - \theta_i)} = \frac{2\cos 19.47^o \sin 90^o}{\sin 19.47^o \cos 19.47^o} = 6$

$$\Gamma_{\parallel} = -\frac{\cot 19.47^o}{\cot 19.47^o} = -1$$

Let $\quad E_t = -E_{to}(\cos\theta_t a_x - \sin\theta_t a_z)\cos(10^9 t - \beta_2 x\sin\theta_t - \beta_2 z\cos\theta_t)$

where

$E_t = -E_{to}(\cos\theta_t a_x - \sin\theta_t a_z)\cos(10^9 t - \beta_1 x\sin\theta_t - \beta_1 z\cos\theta_t)$

$\sin\theta_t = 1, \quad \cos\theta_t = 0, \quad \beta_2 \sin\theta_t = 10/3$

$E_{to}\sin\theta_t = \tau_{//} E_{io} = 6(24\pi)(3)(1) = 1357.2$

Hence,

$\underline{E_t = 1357\cos(10^9 t - 3.333x)a_z}$ V/m

Since $\Gamma = -1, \quad \theta_r = \theta_i$

$\underline{E_r = (213.3a_x + 75.4a_z)\cos(10^9 t - kx + k\sqrt{8}z)}$ V/m

(f) $\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{\varepsilon_o}{9\varepsilon_o}} = 1/3 \quad \longrightarrow \quad \underline{\theta_{B//} = 18.43^o}$

Prob. 10.45

$\beta_1 = \sqrt{3^2 + 4^2} = 5 = \omega / c \quad \longrightarrow \quad \underline{\omega = \beta_1 c = 15x10^8 \text{ rad/s}}$

Let $E_r = (E_{ox}, E_{oy}, E_{oz})\sin(\omega t + 3x + 4y)$. In order for

$\nabla \bullet E_r = 0, \quad 3E_{ox} + 4E_{oy} = 0 \quad (1)$

Also, at y=0, $E_{1tan} = E_{2tan} = 0$

$E_{1tan} = 0, \quad 8a_x + 5a_z + E_{ox}a_x + E_{oz}a_z = 0$

Equating components, $E_{ox} = -8, \quad E_{oz} = -5$

From (1), $4E_{oy} = -3E_{ox} = 24 \quad E_{oy} = 6$

Hence,

$\underline{E_r = (-8a_x + 6a_y - 5a_z)\sin(15x10^8 t + 3x + 4y)}$ V/m

Prob. 10.46 Since both media are nonmagnetic,

$$\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{2.6\varepsilon_o}{\varepsilon_o}} = 1.612 \quad \longrightarrow \quad \theta_{B//} = 58.19^o$$

But

$$\cos\theta_t = \frac{\eta_1}{\eta_2}\cos\theta_{B//} = \frac{\eta_o}{\eta_o/\sqrt{2.6}}\cos\theta_{B//} = \sqrt{2.6}\cos 58.19^o \quad \longrightarrow \quad \underline{\underline{\theta_t = 31.8^o}}$$

where Re signifies that the real part of the following quantity is to be taken. If we then simplify the nomenclature by dropping Re and suppressing $e^{j\omega t}$, the field quantity E_x becomes a phasor, or a complex quantity, which we identify by use of an s subscript, E_{xs}. Thus

$$E_{xs} = E(x, y, z)e^{j\psi} \tag{7}$$

and

$$\mathbf{E_s} = E_{xs}\mathbf{a_x}$$

The s can be thought of as indicating a frequency domain quantity expressed as a function of the complex frequency s, even though we shall consider only those cases in which s is a pure imaginary, $s = j\omega$.

Example 11.1

Let us express $E_y = 100\cos(10^8 t - 0.5z + 30^\circ)$ V/m as a phasor.

Solution. We first go to exponential notation,

$$E_y = \mathrm{Re}[100e^{j(10^8 t - 0.5z + 30^\circ)}]$$

and then drop Re and suppress $e^{j10^8 t}$, obtaining the phasor

$$E_{ys} = 100e^{-j0.5z + j30^\circ}$$

Note that E_y is real, but E_{ys} is in general complex. Note also that a mixed nomenclature is commonly used for the angle. That is, $0.5z$ is in radians, while 30° is in degrees.

Given a scalar component or a vector expressed as a phasor, we may easily recover the time-domain expression.

Example 11.2

Given the field intensity vector, $\mathbf{E_s} = 100\angle 30^\circ \mathbf{a_x} + 20\angle -50^\circ \mathbf{a_y} + 40\angle 210^\circ \mathbf{a_z}$ V/m, identified as a phasor by its subscript s, we desire the vector as a real function of time.

Solution. Our starting point is the phasor,

$$\mathbf{E_s} = 100\angle 30^\circ \mathbf{a_x} + 20\angle -50^\circ \mathbf{a_y} + 40\angle 210^\circ \mathbf{a_z} \text{ V/m}$$

Let us assume that the frequency is specified as 1 MHz. We first select exponential notation for mathematical clarity,

$$\mathbf{E_s} = 100e^{j30^\circ}\mathbf{a_x} + 20e^{-j50^\circ}\mathbf{a_y} + 40e^{j210^\circ}\mathbf{a_z} \text{ V/m}$$

reinsert the $e^{j\omega t}$ factor,

$$\begin{aligned}\mathbf{E_s(t)} &= (100e^{j30^\circ}\mathbf{a_x} + 20e^{-j50^\circ}\mathbf{a_y} + 40e^{j210^\circ}\mathbf{a_z})e^{j2\pi 10^6 t} \\ &= 100e^{j(2\pi 10^6 t + 30^\circ)}\mathbf{a_x} + 20e^{j(2\pi 10^6 t - 50^\circ)}\mathbf{a_y} + 40e^{j(2\pi 10^6 t + 210^\circ)}\mathbf{a_z}\end{aligned}$$

and take the real part, obtaining the real vector,

$$\mathbf{E(t)} = 100\cos(2\pi 10^6 t + 30^\circ)\mathbf{a_x} + 20\cos(2\pi 10^6 t - 50^\circ)\mathbf{a_y} + 40\cos(2\pi 10^6 t + 210^\circ)\mathbf{a_z}$$

None of the amplitudes or phase angles in this example are expressed as a function of x, y, or z, but, if any are, the same procedure is effective. Thus, if $\mathbf{H_s} = 20e^{-(0.1+j20)z}\mathbf{a_x}$ A/m, then

$$\mathbf{H(t)} = \text{Re}[20e^{-0.1z}e^{-j20z}e^{j\omega t}]\mathbf{a_x} = 20e^{-0.1z}\cos(\omega t - 20z)\mathbf{a_x} \text{ A/m}$$

Now, since

$$\frac{\partial E_x}{\partial t} = \frac{\partial}{\partial t}[E(x, y, z)\cos(\omega t + \psi)] = -\omega E(x, y, z)\sin(\omega t + \psi)$$
$$= Re[j\omega E_{xs}e^{j\omega t}]$$

it is evident that taking the partial derivative of any field quantity with respect to time is equivalent to multiplying the corresponding phasor by $j\omega$. As an example, if

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0}\frac{\partial H_y}{\partial z}$$

the corresponding phasor expression is

$$j\omega E_{xs} = -\frac{1}{\epsilon_0}\frac{\partial H_{ys}}{\partial z}$$

where E_{xs} and H_{ys} are complex quantities. We next apply this notation to Maxwell's equations. Thus, given the equation,

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

the corresponding relationship in terms of phasor-vectors is

$$\boxed{\nabla \times \mathbf{H_s} = j\omega\epsilon_0\mathbf{E_s}} \tag{8}$$

Equation (8) and the three equations

$$\nabla \times \mathbf{E_s} = -j\omega\mu_0\mathbf{H_s} \tag{9}$$
$$\nabla \cdot \mathbf{E_s} = 0 \tag{10}$$
$$\nabla \cdot \mathbf{H_s} = 0 \tag{11}$$

are Maxwell's four equations in phasor notation for sinusoidal time variation in free space. It should be noted that (10) and (11) are no longer independent relationships, for they can be obtained by taking the divergence of (8) and (9), respectively.

The wavelength is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{c}{f\sqrt{\mu_R\epsilon_R}} = \frac{\lambda_0}{\sqrt{\mu_R\epsilon_R}} \qquad \text{(lossless medium)} \tag{42}$$

where λ_0 is the free space wavelength. Note that $\mu_R\epsilon_R > 1$, and therefore the wavelength is shorter and the velocity is lower in all real media than they are in free space.

Associated with E_x is the magnetic field intensity

$$H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)$$

where the intrinsic impedance is

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \tag{43}$$

The two fields are once again perpendicular to each other, perpendicular to the direction of propagation, and in phase with each other everywhere. Note that when **E** is crossed into **H**, the resultant vector is in the direction of propagation. We shall see the reason for this when we discuss the Poynting vector.

Example 11.3

Let us apply these results to a 1 MHz plane wave propagating in fresh water. At this frequency, losses in water are known to be small, so for simplicity, we will neglect ϵ''. In water, $\mu_R = 1$ and at 1 MHz, $\epsilon'_R = \epsilon_R = 81$.

Solution. We begin by calculating the phase constant. Using (36) with $\epsilon'' = 0$, we have

$$\beta = \omega\sqrt{\mu\epsilon'} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon'_R} = \frac{\omega\sqrt{\epsilon'_R}}{c} = \frac{2\pi \times 10^6\sqrt{81}}{3.0 \times 10^8} = 0.19 \text{ rad/m}$$

Using this result, we can determine the wavelength and phase velocity:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{.19} = 33 \text{ m}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{.19} = 3.3 \times 10^7 \text{ m/s}$$

The wavelength in air would have been 300 m. Continuing our calculations, we find the intrinsic impedance, using (39) with $\epsilon'' = 0$:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} = \frac{\eta_0}{\sqrt{\epsilon'_R}} = \frac{377}{9} = 42\ \Omega$$

If we let the electric field intensity have a maximum amplitude of 0.1 V/m, then

$$E_x = 0.1\cos(2\pi 10^6 t - .19z)\ \text{V/m}$$

$$H_y = \frac{E_x}{\eta} = 2.4 \times 10^{-3}\cos(2\pi 10^6 t - .19z)\ \text{A/m}$$

✔ **D11.3.** A 9.375-GHz uniform plane wave is propagating in polyethylene (see Appendix C). If the amplitude of the electric field intensity is 500 V/m and the material is assumed to be lossless, find: (a) the phase constant; (b) the wavelength in the polyethylene; (c) the velocity of propagation; (d) the intrinsic impedance; (e) the amplitude of the magnetic field intensity.

Ans. 295 rad/m; 2.13 cm; 1.99×10^8 m/s; 251 Ω; 1.99 A/m

Example 11.4

We again consider plane wave propagation in water, but at the much higher microwave frequency of 2.5 GHz. At frequencies in this range and higher, dipole relaxation and resonance phenomena[3] in the water molecules become important. Real and imaginary parts of the permittivity are present, and both vary with frequency. At frequencies below that of visible light, the two mechanisms together produce an ϵ'' that increases with increasing frequency, reaching a local maximum in the vicinity of 10^{10} Hz. ϵ' decreases with increasing frequency. Ref. 3 provides specific details. At 2.5 GHz, dipole relaxation effects dominate. The permittivity values are $\epsilon'_R = 78$ and $\epsilon''_R = 7$. From(35), we have

$$\alpha = \frac{(2\pi \times 2.5 \times 10^9)\sqrt{78}}{(3.0 \times 10^8)\sqrt{2}}\left(\sqrt{1 + \left(\frac{7}{78}\right)^2} - 1\right)^{1/2} = 21\ \text{Np/m}$$

The first calculation demonstrates the operating principle of the *microwave oven*. Almost all foods contain water, and so can be cooked when incident microwave radiation is absorbed and converted into heat. Note that the field will attenuate to a value of e^{-1} times its initial value at a distance of $1/\alpha = 4.8$ cm. This distance is called the *penetration depth* of the material, and of course is frequency-dependent. The 4.8 cm depth is reasonable for cooking food, since it would lead to a temperature rise that is fairly uniform throughout the depth of the material. At much higher frequencies, where ϵ'' is larger, the penetration depth decreases, and too much power is absorbed at the surface; at lower frequencies, the penetration depth increases, and not enough overall absorption occurs. Commercial microwave ovens operate at frequencies in the vicinity of 2.5 GHz.

Using (36), in a calculation very similar to that for α, we find $\beta = 464$ rad/m. The wavelength is $\lambda = 2\pi/\beta = 1.4$ cm, whereas in free space this would have been $\lambda_0 = c/f = 12$ cm.

[3] These mechanisms and how they produce a complex permittivity are described in Appendix D. Additionally, the reader is referred to pp. 73–84 in Ref. 1 and pp. 678–682 in Ref. 2 for general treatments of relaxation and resonance effects on wave propagation. Discussions and data that are specific to water are presented in Ref. 3, pp. 314–316.

Using (39), the intrinsic impedance is found to be

$$\eta = \frac{377}{\sqrt{78}} \frac{1}{\sqrt{1 - j(7/78)}} = 43 + j1.9 = 43\angle 2.6^\circ \ \Omega$$

and E_x leads H_y in time by 2.6° at every point.

We next consider the case of conductive materials, in which currents are formed by the motion of free electrons or holes under the influence of an electric field. The governing relation is $\mathbf{J} = \sigma\mathbf{E}$, where σ is the material conductivity. With finite conductivity, the wave loses power through resistive heating of the material.We look for an interpretation of the complex permittivity as it relates to the conductivity. Consider the Maxwell curl equation (8) which, using (33), becomes:

$$\nabla \times \mathbf{H}_s = j\omega(\epsilon' - j\epsilon'')\mathbf{E}_s = \omega\epsilon''\mathbf{E}_s + j\omega\epsilon'\mathbf{E}_s \tag{44}$$

This equation can be expressed in a more familiar way, in which conduction current is included:

$$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega\epsilon\mathbf{E}_s \tag{45}$$

We next use $\mathbf{J}_s = \sigma\mathbf{E}_s$, and interpret ϵ in (41) as ϵ'. Eq. (45) then becomes:

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\epsilon')\mathbf{E}_s = \mathbf{J}_{\sigma s} + \mathbf{J}_{ds} \tag{46}$$

which we have expressed in terms of conduction current density, $\mathbf{J}_{\sigma s} = \sigma\mathbf{E}_s$, and displacement current density, $\mathbf{J}_{ds} = j\omega\epsilon'\mathbf{E}_s$. Comparing Eqs. (44) and (46), we find that in a conductive medium:

$$\boxed{\epsilon'' = \frac{\sigma}{\omega}} \tag{47}$$

Let us now turn our attention to the case of a dielectric material in which the loss is very small. The criterion by which we would judge whether or not the loss is small is the magnitude of the loss tangent, ϵ''/ϵ'. This parameter will have a direct influence on the attenuation coefficient, α, as seen from Eq. (35). In the case of conducting media in which (47) holds, the loss tangent becomes $\sigma/\omega\epsilon'$. By inspecting (46), we see that the ratio of condution current density to displacement current density magnitudes is

$$\frac{J_{\sigma s}}{J_{ds}} = \frac{\epsilon''}{j\epsilon'} = \frac{\sigma}{j\omega\epsilon'} \tag{48}$$

That is, these two vectors point in the same direction in space, but they are 90° out of phase in time. Displacement current density leads conduction current density by 90°, just as the current through a capacitor leads the current through a resistor in parallel with it by 90° in an ordinary electric current. This phase relationship is shown in Fig. 11.2. The angle θ (not to be confused with the polar

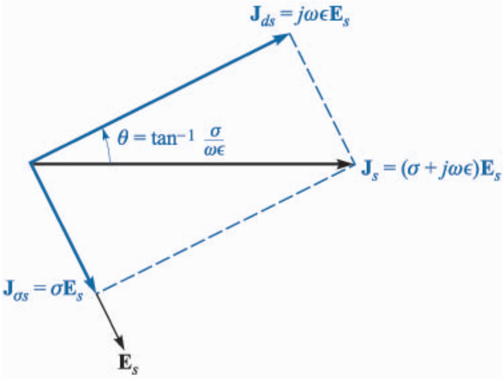


FIGURE 11.2
The time-phase relationship between $\mathbf{J}_{ds}$, $\mathbf{J}_{\sigma s}$, $\mathbf{J}_s$, and $\mathbf{E}_s$. The tangent of θ is equal to $\sigma/\omega\epsilon$, and $90^\circ - \theta$ is the common power-factor angle, or the angle by which $\mathbf{J}_s$ leads $\mathbf{E}_s$.

angle in spherical coordinates) may therefore be identified as the angle by which the displacement current density leads the total current density, and

$$\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon'} \tag{49}$$

The reasoning behind the term "loss tangent" is thus evident. Problem 16 at the end of the chapter indicates that the Q of a capacitor (its quality factor, not its charge) which incorporates a lossy dielectric is the reciprocal of the loss tangent.

If the loss tangent is small, then we may obtain useful approximations for the attenuation and phase constants, and the intrinsic impedance. Considering a conductive material, for which $\epsilon'' = \sigma/\omega$, (34) becomes

$$jk = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}} \tag{50}$$

We may expand the second radical using the binomial theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{(2!)}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

where $|x| \ll 1$. We identify x as $-j\sigma/\omega\epsilon'$ and n as $1/2$, and thus

$$jk = j\omega\sqrt{\mu\epsilon'}\left[1 - j\frac{\sigma}{2\omega\epsilon'} + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon'}\right)^2 + \ldots\right] = \alpha + j\beta$$

Now

$$\alpha = \mathrm{Re}(jk) \doteq j\omega\sqrt{\mu\epsilon'}\left(-j\frac{\sigma}{2\omega\epsilon'}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon'}} \tag{51}$$

and

$$\beta = \mathrm{Im}(jk) \doteq \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon'}\right)^2\right] \qquad (52a)$$

or in many cases

$$\beta \doteq \omega\sqrt{\mu\epsilon'} \qquad (52b)$$

Applying the binomial expansion to (39), we obtain

$$\eta \doteq \sqrt{\frac{\mu}{\epsilon'}}\left[1 - \frac{3}{8}\left(\frac{\sigma}{\omega\epsilon'}\right)^2 + j\frac{\sigma}{2\omega\epsilon'}\right] \qquad (53a)$$

or

$$\eta \doteq \sqrt{\frac{\mu}{\epsilon'}}\left(1 + j\frac{\sigma}{2\omega\epsilon'}\right) \qquad (53b)$$

The conditions under which the above approximations can be used depend on the desired accuracy, measured by how much the results deviate from those given by the exact formulas, (35) and (36). Deviations of no more than a few percent occur if $\sigma/\omega\epsilon' < 0.1$.

Example 11.5

As a comparison, we repeat the computations of Example 11.4, using the approximation formulas, (51), (52*b*), and (53*b*).

Solution. First, the loss tangent in this case is $\epsilon''/\epsilon' = 7/78 = 0.09$. Using (51), with $\epsilon'' = \sigma/\omega$, we have

$$\alpha \doteq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{1}{2}(7 \times 8.85 \times 10^{12})(2\pi \times 2.5 \times 10^9)\frac{377}{\sqrt{78}} = 21 \text{ cm}^{-1}$$

We then have, using (52*b*),

$$\beta \doteq (2\pi \times 2.5 \times 10^9)\sqrt{78}/(2.99 \times 10^8) = 464 \text{ rad/m}$$

Finally, with (53*b*),

$$\eta \doteq \frac{377}{\sqrt{78}}\left(1 + j\frac{7}{2 \times 78}\right) = 43 + j1.9$$

These results are identical (within the accuracy limitations as determined by the given numbers) to those of Example 11.4. Small deviations will be found, as the reader can verify by repeating the calculations of both examples and expressing the results to four or five significant figures. As we know, this latter practice would not be meaningful since the given parameters were not specified with such accuracy. Such is often the case, thickness of about 1/2 in would be a much better design. Although we are applying the results of an analysis for an infinite planar conductor to one of finite dimensions, the fields are attenuated in the finite-size conductor in a similar (but not identical) fashion.

The extremely short skin depth at microwave frequencies shows that only the surface coating of the guiding conductor is important. A piece of glass with an evaporated silver surface 0.0001 in thick is an excellent conductor at these frequencies.

Next, let us determine expressions for the velocity and wavelength within a good conductor. From (62), we already have

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu \sigma}$$

Then, since

$$\beta = \frac{2\pi}{\lambda}$$

we find the wavelength to be

$$\boxed{\lambda = 2\pi\delta} \tag{63}$$

Also, recalling that

$$v_p = \frac{\omega}{\beta}$$

we have

$$\boxed{v_p = \omega\delta} \tag{64}$$

For copper at 60 Hz, $\lambda = 5.36$ cm and $v_p = 3.22$ m/s, or about 7.2 mi/h. A lot of us can run faster than that. In free space, of course, a 60-Hz wave has a wavelength of 3100 mi and travels at the velocity of light.

Example 11.6

Let us again consider wave propagation in water, but this time we will consider seawater. The primary difference between seawater and fresh water is of course the salt content. Sodium chloride dissociates in water to form Na^+ and Cl^- ions, which, being charged, will move when forced by an electric field. Seawater is thus conductive, and so will attenuate electromagnetic waves by this mechanism. At frequencies in the vicinity of 10^7 Hz and below, the bound charge effects in water discussed earlier are negligible, and losses in seawater arise principally from the salt-associated conductivity. We consider an incident wave of frequency 1 MHz. We wish to find the skin depth, wavelength, and phase velocity. In seawater, $\sigma = 4$ S/m, and $\epsilon'_R = 81$.

Solution. We first evaluate the loss tangent, using the given data:

$$\frac{\sigma}{\omega\epsilon'} = \frac{4}{(2\pi \times 10^6)(81)(8.85 \times 10^{-12})} = 8.9 \times 10^2 \gg 1$$

Thus, seawater is a good conductor at 1 MHz (and at frequencies lower than this). The skin depth is

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{(\pi \times 10^6)(4\pi \times 10^{-7})(4)}} = 0.25 \text{ m} = 25 \text{ cm}$$

Now

$$\lambda = 2\pi\delta = 1.6 \text{ m}$$

and

$$v_p = \omega\delta = (2\pi \times 10^6)(0.25) = 1.6 \times 10^6 \text{ m/sec}$$

In free space, these values would have been $\lambda = 300$ m and of course $v = c$.

With a 25 cm skin depth, it is obvious that radio frequency communication in seawater is quite impractical. Notice however that δ varies as $1/\sqrt{f}$, so that things will improve at lower frequencies. For example, if we use a frequency of 10 Hz in the extremely low frequency (ELF) range, the skin depth is increased over that at 1 MHz by a factor of $\sqrt{10^6/10}$, so that

$$\delta(10Hz) \doteq 80 \text{ m}$$

The corresponding wavelength is $\lambda = 2\pi\delta \doteq 500$ m. Frequencies in the ELF range are in fact used for submarine communications, chiefly between gigantic ground-based antennas (required since the free-space wavelength associated with 10 Hz is 3×10^7 m) and submarines, from which a suspended wire antenna of length shorter than 500 m is sufficient to receive the signal. The drawback is that signal data rates at ELF are so slow that a single word can take several minutes to transmit. Typically, ELF signals are used to tell the submarine to implement emergency procedures, or to come near the surface in order to receive a more detailed message via satellite.

We next turn our attention to finding the magnetic field, H_y, associated with E_x. To do so, we need an expression for the intrinsic impedance of a good conductor. We begin with Eq. (39), Sec. 11.2, with $\epsilon'' = \sigma/\omega$,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon'}}$$

Since $\sigma \gg \omega\epsilon'$, we have

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

which may be written as

$$\boxed{\eta = \frac{\sqrt{2}\angle 45^\circ}{\sigma\delta} = \frac{1}{\sigma\delta} + j\frac{1}{\sigma\delta}} \qquad (65)$$

Example 12.3

A uniform plane wave in air partially reflects from the surface of a material whose properties are unknown. Measurements of the electric field in the region in front of the interface yield a 1.5 m spacing between maxima, with the first maximum occurring 0.75 m from the interface. A standing wave ratio of 5 is measured. Determine the intrinsic impedance, η_u, of the unknown material.

Solution. The 1.5 m spacing between maxima is $\lambda/2$, implying a wavelength is 3.0 m, or $f = 100$ MHz. The first maximum at 0.75 m is thus at a distance of $\lambda/4$ from the interface, which means that a field minimum occurs at the boundary. Thus Γ will be real and negative. We use (27) to write

$$|\Gamma| = \frac{s-1}{s+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

So

$$\Gamma = -\frac{2}{3} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

which we solve for η_u to obtain

$$\eta_u = \frac{1}{5}\eta_0 = \frac{377}{5} = 75.4\,\Omega$$

12.3 WAVE REFLECTION FROM MULTIPLE INTERFACES

So far we have treated the reflection of waves at the single boundary that occurs between semi-infinite media. In this section, we consider wave reflection from materials that are finite in extent, such that we must consider the effect of the front and back surfaces. Such a two-interface problem would occur, for example, for light incident on a flat piece of glass. Additional interfaces are present if the glass is coated with one or more layers of dielectric material for the purpose (as we will see) of reducing reflections. Such problems in which more than one interface is involved are frequently encountered; single interface problems are in fact more the exception than the rule.

Consider the general situation shown in Fig. 12.6, in which a uniform plane wave propagating in the forward z direction is normally incident from the left onto the interface between regions 1 and 2; these have intrinsic impedances, η_1 and η_2. A third region of impedance η_3 lies beyond region 2, and so a second interface exists between regions 2 and 3. We let the second interface location occur at $z = 0$, and so all positions to the left will be described by values of z that are negative. The width of the second region is l, so the first interface will occur at position $z = -l$.

When the incident wave reaches the first interface, events occur as follows: A portion of the wave reflects, while the remainder is transmitted, to propagate toward the second interface. There, a portion is transmitted into region 3, while

With the conditions given by (44) and (46) satisfied, we have performed *quarter-wave matching*. The design of anti reflective coatings for optical devices is based on this principle.

Example 12.5

We wish to coat a glass surface with an appropriate dielectric layer to provide total transmission from air to the glass at a wavelength of 570 nm. The glass has dielectric constant, $\epsilon_R = 2.1$. Determine the required dielectric constant for the coating and its minimum thickness.

Solution. The known impedances are $\eta_1 = 377\,\Omega$ and $\eta_3 = 377/\sqrt{2.1} = 260\,\Omega$. Using (46) we have

$$\eta_2 = \sqrt{(377)(260)} = 313\,\Omega$$

The dielectric constant of region 2 will then be

$$\epsilon_{R2} = \left(\frac{377}{313}\right)^2 = 1.45$$

The wavelength in region 2 will be

$$\lambda_2 = \frac{570}{\sqrt{1.45}} = 473\,\text{nm}$$

The minimum thickness of the dielectric layer is then

$$l = \frac{\lambda_2}{4} = 118\,\text{nm} = 0.118\,\mu\text{m}$$

The procedure in this section for evaluating wave reflection has involved calculating an effective impedance at the first interface, η_{in}, which is expressed in terms of the impedances that lie beyond the front surface. This process of *impedance transformation* is more apparent when we consider problems involving more than two interfaces.

For example, consider the three-interface situation shown in Fig. 12.7, where a wave is incident from the left in region 1. We wish to determine the fraction of the incident power that is reflected and back-propagates in region 1, and the fraction of the incident power that is transmitted into region 4. To do this, we need to find the input impedance at the front surface (the interface between regions 1 and 2). We start by transforming the impedance of region 4 to form the input impedance at the boundary between regions 2 and 3. This is shown as $\eta_{in,b}$ in the figure. Using (41), we have

$$\eta_{in,b} = \eta_3 \frac{\eta_4 \cos \beta_3 l_b + j\eta_3 \sin \beta_3 l_b}{\eta_3 \cos \beta_3 l_b + j\eta_4 \sin \beta_3 l_b} \tag{47}$$

Example 12.6

Consider a 50 MHz uniform plane wave having electric field amplitude 10 V/m. The medium is lossless, having $\epsilon_R = \epsilon'_R = 9.0$ and $\mu_R = 1.0$. The wave propagates in the x, y plane at a 30° angle to the x axis, and is linearly polarized along z. Write down the phasor expression for the electric field.

Solution. The propagation constant magnitude is

$$k = \omega\sqrt{\mu\epsilon} = \frac{\omega\sqrt{\epsilon_R}}{c} = \frac{2\pi \times 50 \times 10^6(3)}{3 \times 10^8} = 3.14\,\text{m}^{-1}$$

The vector **k** is now

$$\mathbf{k} = 3.14(\cos 30\,\mathbf{a_x} + \sin 30\,\mathbf{a_y}) = 2.7\mathbf{a_x} + 1.6\mathbf{a_y}\,\text{m}^{-1}$$

Then

$$\mathbf{r} = x\,\mathbf{a_x} + y\,\mathbf{a_y}$$

With the electric field directed along z, the phasor form becomes

$$\mathbf{E_s} = E_0 e^{-j\mathbf{k}\cdot\mathbf{r}}\,\mathbf{a_z} = 10e^{-j(2.7x+1.6y)}\,\mathbf{a_z}$$

✔ **D12.4.** For Example 12.6, calculate λ_x, λ_y, v_{px}, and v_{py}.

Ans. 2.3 m; 3.9 m; 1.2×10^8 m/s; 2.0×10^8 m/s.

12.5 PLANE WAVE REFLECTION AT OBLIQUE INCIDENCE ANGLES

We now consider the problem of wave reflection from plane interfaces, in which the incident wave propagates at some angle to the surface. Our objectives are (1) to determine the relation between incident, reflected, and transmitted angles, and (2) to derive reflection and transmission coefficients that are functions of the incident angle and wave polarization. We will also show that cases exist in which total reflection or total transmission may occur at the interface between two dielectrics if the angle of incidence and the polarization are appropriately chosen.

The situation is illustrated in Fig. 12.9, in which the incident wave direction and position-dependent phase are characterized by wavevector, $\mathbf{k}_1^+$. The angle of incidence is the angle between $\mathbf{k}_1^+$ and a line that is normal to the surface (the x axis in this case). The incidence angle is shown as θ_1. The reflected wave, characterized by wavevector $\mathbf{k}_1^-$, will propagate away from the interface at angle θ'_1. Finally, the transmitted wave, characterized by $\mathbf{k}_2$, will propagate into the second region at angle θ_2 as shown. One would suspect (from previous experience) that the incident and reflected angles are equal ($\theta_1 = \theta'_1$), which is correct. We need to show this, however, to be complete.

The two media are lossless dielectrics, characterized by intrinsic impedances, η_1 and η_2. We will assume, as before, that the materials are non-where we have used $\eta_1 = \eta_0/n_1$ and $\eta_2 = \eta_0/n_2$. We call this special angle, θ_B, where total transmission occurs, the *Brewster angle* or *polarization angle*. The latter name comes from the fact that if light having both s- and p-polarization components is incident at $\theta_1 = \theta_B$, the p component will be totally transmitted, leaving the partially reflected light entirely s-polarized. At angles that are slightly off the Brewster angle, the reflected light is still predominantly s-polarized. Most reflected light that we see originates from horizontal surfaces (such as the surface of the ocean), and as such, the light is mostly of horizontal polarization. Polaroid sunglasses take advantage of this fact to reduce glare, since they are made to block transmission of horizontally polarized light, while passing light that is vertically polarized.

Example 12.9

Light is incident from air to glass at Brewster's angle. Determine the incident and transmitted angles.

Solution. Since glass has refractive index $n_2 = 1.45$, the incident angle will be

$$\theta_1 = \theta_B = \sin^{-1}\left(\frac{n_2}{\sqrt{n_1^2 + n_2^2}}\right) = \sin^{-1}\left(\frac{1.45}{\sqrt{1.45^2 + 1}}\right) = 55.4^\circ$$

The transmitted angle is found from Snell's law, through

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_B\right) = \sin^{-1}\left(\frac{n_1}{\sqrt{n_1^2 + n_2^2}}\right) = 34.6^\circ$$

Note from this exercise that $\sin\theta_2 = \cos\theta_B$, which means that the sum of the incident and refracted angles at the Brewster condition is always 90°.

✔ **D12.5.** In Example 12.9, calculate the reflection coefficient for s-polarized light.

Ans. −0.355

Many of the results we have seen in this section are summarized in Fig. 12.12, in which Γ_p and Γ_s, from (69) and (71), are plotted as functions of the incident angle, θ_1. Curves are shown for selected values of the refractive index ratio, n_1/n_2. For all plots in which $n_1/n_2 > 1$, Γ_s and Γ_p achieve a value of ± 1 at the critical angle. At larger angles, the reflection coefficients become imaginary (and are not shown) but nevertheless retain magnitudes of unity. The occurrence of the Brewster angle is evident in the curves for Γ_p (Fig. 12.12*a*), as all curves cross the θ_1 axis. This behavior is not seen in the Γ_s functions (Fig. 12.12*b*), as Γ_s is positive for all values of θ_1 when $n_1/n_2 > 1$, and is negative for $n_1/n_2 < 1$.

Referring to the ω-β diagram, Fig. 12.14, we recognize the carrier phase velocity as the slope of the straight line that joins the origin to the point on the curve whose coordinates are ω_0 and β_0. We recognize the envelope velocity as a quantity that approximates the slope of the ω-β curve at the location of an operation point specified by (ω_0, β_0). The envelope velocity in this case is thus somewhat less than the carrier velocity. As $\Delta\omega$ becomes vanishingly small, the envelope velocity is identically the slope of the curve at ω_0. We can thus state the following for our example:

$$\lim_{\Delta\omega \to 0} \frac{\Delta\omega}{\Delta\beta} = \left.\frac{d\omega}{d\beta}\right|_{\omega_0} = v_g(\omega_0) \tag{85}$$

The quantity $d\omega/d\beta$ is called the *group velocity* function for the material, $v_g(\omega)$. When evaluated at a specified frequency, ω_0, it represents the velocity of a group of frequencies within a spectral packet of vanishingly small width, centered at frequency ω_0. In stating this, we have extended our two-frequency example to include waves that have a continuous frequency spectrum. To each frequency component (or packet) is associated a group velocity at which the energy in that packet propagates. Since the slope of the ω-β curve changes with frequency, group velocity will obviously be a function of frequency. The *group velocity dispersion* of the medium is, to first order, the rate at which the slope of the ω-β curve changes with frequency. It is this behavior that is of critical practical importance to the propagation of modulated waves within dispersive media, and the extent to which the modulation envelope may degrade with propagation distance.

Example 12.10

Consider a medium in which the refractive index varies linearly with frequency over a certain range:

$$n(\omega) = n_0 \frac{\omega}{\omega_0}$$

Determine the group velocity and the phase velocity of a wave at frequency ω_0.

Solution. First, the phase constant will be

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \frac{n_0\omega^2}{\omega_0 c}$$

Now

$$\frac{d\beta}{d\omega} = \frac{2n_0\omega}{\omega_0 c}$$

so that

$$v_g = \frac{d\omega}{d\beta} = \frac{\omega_0 c}{2n_0\omega}$$

Example 12.11

An optical fiber channel is known to have dispersion, $\beta_2 = 20\ \text{ps}^2/\text{km}$. A Gaussian light pulse at the input of the fiber is of initial width $T = 10$ ps. Determine the width of the pulse at the fiber output, if the fiber is 15 km long.

Solution. The pulse spread will be

$$\Delta\tau = \frac{\beta_2 z}{T} = \frac{(20)(15)}{10} = 30\ \text{ps}$$

So the output pulse width is

$$T' = \sqrt{(10)^2 + (30)^2} = 32\ \text{ps}$$

An interesting by-product of pulse broadening through chromatic dispersion is that the broadened pulse is *chirped*. This means that the instantaneous frequency of the pulse varies monotonically (either increases or decreases) with time over the pulse envelope. This again is just a manifestation of the broadening mechanism, in which the spectral components at different frequencies are spread out in time as they propagate at different group velocities. We can quantify the effect by calculating the group delay, τ_g, as a function of frequency, using (92). We obtain:

$$\tau_g = \frac{z}{v_g} = z\frac{d\beta}{d\omega} = (\beta_1 + (\omega - \omega_0)\beta_2)z \tag{95}$$

This equation tells us that the group delay will be a linear function of frequency, and that higher frequencies will arrive at later times, if β_2 is positive. We refer to the chirp as positive if lower frequencies lead the higher frequencies in time (requiring a positive β_2 in (95)); chirp is negative if the higher frequencies lead in time (negative β_2). Fig. 12.17 shows the broadening effect and illustrates the chirping phenomenon.

✔ **D12.6.** For the fiber channel of Example 12.11, a 20 ps pulse is input instead of the 10 ps pulse in the example. Determine the output pulsewidth.

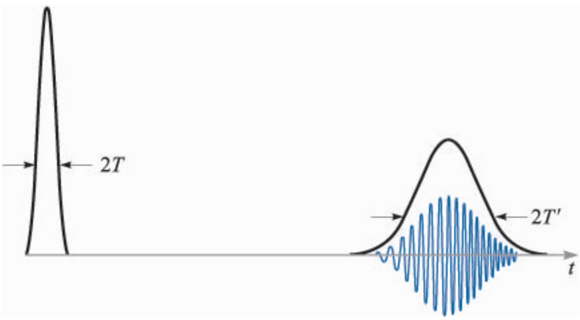


FIGURE 12.17
Gaussian pulse intensities as functions of time (smooth curves) before and after propagation through a dispersive medium, as exemplified by the ω-β diagram of Fig. 12.16*b*. The electric field oscillations are shown under the second trace to demonstrate the chirping effect as the pulse broadens. Note the reduced amplitude of the broadened pulse, which occurs because the pulse energy (the area under the intensity envelope) is constant.

CHAPTER 11

11.1. Show that $E_{xs} = Ae^{jk_0z+\phi}$ is a solution to the vector Helmholtz equation, Sec. 11.1, Eq. (16), for $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ and any ϕ and A: We take

$$\frac{d^2}{dz^2} Ae^{jk_0z+\phi} = (jk_0)^2 Ae^{jk_0z+\phi} = -k_0^2 E_{xs}$$

11.2. Let $\mathbf{E}(z,t) = 200 \sin 0.2z \cos 10^8 t \mathbf{a}_x + 500 \cos(0.2z + 50^\circ) \sin 10^8 t \mathbf{a}_y$ V/m. Find:

a) $\mathbf{E}$ at $P(0, 2, 0.6)$ at $t = 25$ ns: Obtain

$$\begin{aligned} E_P(t=25) &= 200 \sin[(0.2)(0.6)] \cos(2.5)\mathbf{a}_x + 500 \cos[(0.2)(0.6) + 50(2\pi)/360] \sin(2.5)\mathbf{a}_y \\ &= \underline{-19.2\mathbf{a}_x + 164\mathbf{a}_y \text{ V/m}} \end{aligned}$$

b) $|\mathbf{E}|$ at P at $t = 20$ ns:

$$\begin{aligned} E_P(t=20) &= 200 \sin[(0.2)(0.6)] \cos(2.0)\mathbf{a}_x + 500 \cos[(0.2)(0.6) + 50(2\pi)/360] \sin(2.0)\mathbf{a}_y \\ &= -9.96\mathbf{a}_x + 248\mathbf{a}_y \text{ V/m} \end{aligned}$$

Thus $|\mathbf{E}_P| = \sqrt{(9.96)^2 + (248)^2} = \underline{249 \text{ V/m}}$.

c) E_s at P: $E_s = 200 \sin 0.2z\mathbf{a}_x - j500 \cos(0.2z + 50^\circ)\mathbf{a}_y$. Thus

$$\begin{aligned} E_{sP} &= 200 \sin[(0.2)(0.6)]\,\mathbf{a}_x - j500 \cos[(0.2)(0.6) + 2\pi(50)/360]\,\mathbf{a}_y \\ &= \underline{23.9\mathbf{a}_x - j273\mathbf{a}_y \text{ V/m}} \end{aligned}$$

11.3. An $\mathbf{H}$ field in free space is given as $\mathbf{H}(x,t) = 10 \cos(10^8 t - \beta x)\mathbf{a}_y$ A/m. Find

a) β: Since we have a uniform plane wave, $\beta = \omega/c$, where we identify $\omega = 10^8$ sec^{-1}. Thus $\beta = 10^8/(3 \times 10^8) = \underline{0.33 \text{ rad/m}}$.

b) λ: We know $\lambda = 2\pi/\beta = \underline{18.9 \text{ m}}$.

c) $\mathbf{E}(x,t)$ at $P(0.1, 0.2, 0.3)$ at $t = 1$ ns: Use $E(x,t) = -\eta_0 H(x,t) = -(377)(10)\cos(10^8 t - \beta x) = -3.77 \times 10^3 \cos(10^8 t - \beta x)$. The vector direction of $\mathbf{E}$ will be $-\mathbf{a}_z$, since we require that $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, where $\mathbf{S}$ is x-directed. At the given point, the relevant coordinate is $x = 0.1$. Using this, along with $t = 10^{-9}$ sec, we finally obtain

$$\begin{aligned} \mathbf{E}(x,t) &= -3.77 \times 10^3 \cos[(10^8)(10^{-9}) - (0.33)(0.1)]\mathbf{a}_z = -3.77 \times 10^3 \cos(6.7 \times 10^{-2})\mathbf{a}_z \\ &= \underline{-3.76 \times 10^3 \mathbf{a}_z \text{ V/m}} \end{aligned}$$

11.4. In phasor form, the electric field intensity of a uniform plane wave in free space is expressed as $\mathbf{E}_s = (40 - j30)e^{-j20z}\mathbf{a}_x$ V/m. Find:

a) ω: From the given expression, we identify $\beta = 20$ rad/m. Then $\omega = c\beta = (3 \times 10^8)(20) = \underline{6.0 \times 10^9 \text{ rad/s}}$.

b) $\beta = \underline{20 \text{ rad/m}}$ from part a.

11.4. (continued)

c) $f = \omega/2\pi = \underline{956 \text{ MHz}}$.

d) $\lambda = 2\pi/\beta = 2\pi/20 = \underline{0.314 \text{ m}}$.

e) $\mathbf{H}_s$: In free space, we find $\mathbf{H}_s$ by dividing $\mathbf{E}_s$ by η_0, and assigning vector components such that $\mathbf{E}_s \times \mathbf{H}_s$ gives the required direction of wave travel: We find

$$\mathbf{H}_s = \frac{40 - j30}{377} e^{-j20z} \mathbf{a}_y = \underline{(0.11 - j0.08)e^{-j20z} \, \mathbf{a}_y \text{ A/m}}$$

f) $\mathbf{H}(z, t)$ at $P(6, -1, 0.07)$, $t = 71$ ps:

$$\mathbf{H}(z,t) = \text{Re}\left[\mathbf{H}_s e^{j\omega t}\right] = \left[0.11 \cos(6.0 \times 10^9 t - 20z) + 0.08 \sin(6.0 \times 10^9 t - 20z)\right] \mathbf{a}_y$$

Then

$$\begin{aligned} \mathbf{H}(.07, t = 71\text{ps}) &= \Big[0.11 \cos\left[(6.0 \times 10^9)(7.1 \times 10^{-11}) - 20(.07)\right] \\ &\quad + .08 \sin\left[(6.0 \times 10^9)(7.1 \times 10^{-11}) - 20(.07)\right]\Big] \mathbf{a}_y \\ &= [0.11(0.562) - 0.08(0.827)]\mathbf{a}_y = \underline{-6.2 \times 10^{-3} \mathbf{a}_y \text{ A/m}} \end{aligned}$$

11.5. A 150-MHz uniform plane wave in free space is described by $\mathbf{H}_s = (4 + j10)(2\mathbf{a}_x + j\mathbf{a}_y)e^{-j\beta z}$ A/m.

a) Find numerical values for ω, λ, and β: First, $\omega = 2\pi \times 150 \times 10^6 = \underline{3\pi \times 10^8 \text{ sec}^{-1}}$. Second, for a uniform plane wave in free space, $\lambda = 2\pi c/\omega = c/f = (3 \times 10^8)/(1.5 \times 10^8) = \underline{2 \text{ m}}$. Third, $\beta = 2\pi/\lambda = \underline{\pi \text{ rad/m}}$.

b) Find $\mathbf{H}(z, t)$ at $t = 1.5$ ns, $z = 20$ cm: Use

$$\begin{aligned} \mathbf{H}(z,t) &= \text{Re}\{\mathbf{H}_s e^{j\omega t}\} = \text{Re}\{(4 + j10)(2\mathbf{a}_x + j\mathbf{a}_y)(\cos(\omega t - \beta z) + j \sin(\omega t - \beta z)\} \\ &= [8\cos(\omega t - \beta z) - 20\sin(\omega t - \beta z)]\, \mathbf{a}_x - [10\cos(\omega t - \beta z) + 4\sin(\omega t - \beta z)]\, \mathbf{a}_y \end{aligned}$$

. Now at the given position and time, $\omega t - \beta z = (3\pi \times 10^8)(1.5 \times 10^{-9}) - \pi(0.20) = \pi/4$. And $\cos(\pi/4) = \sin(\pi/4) = 1/\sqrt{2}$. So finally,

$$\mathbf{H}(z = 20\text{cm}, t = 1.5\text{ns}) = -\frac{1}{\sqrt{2}}\left(12\mathbf{a}_x + 14\mathbf{a}_y\right) = \underline{-8.5\mathbf{a}_x - 9.9\mathbf{a}_y \text{ A/m}}$$

c) What is $|E|_{max}$? Have $|E|_{max} = \eta_0 |H|_{max}$, where

$$|H|_{max} = \sqrt{\mathbf{H}_s \cdot \mathbf{H}_s^*} = [4(4 + j10)(4 - j10) + (j)(-j)(4 + j10)(4 - j10)]^{1/2} = 24.1 \text{ A/m}$$

Then $|E|_{max} = 377(24.1) = \underline{9.08 \text{ kV/m}}$.

11.6. Let $\mu_R = \epsilon_R = 1$ for the field $\mathbf{E}(z,t) = (25\mathbf{a}_x - 30\mathbf{a}_y)\cos(\omega t - 50z)$ V/m.

a) Find ω: $\omega = c\beta = (3 \times 10^8)(50) = \underline{15.0 \times 10^9\ \text{s}^{-1}}$.

b) Determine the displacement current density, $\mathbf{J}_d(z,t)$:

$$\mathbf{J}_d(z,t) = \frac{\partial \mathbf{D}}{\partial t} = -\epsilon_0\omega(25\mathbf{a}_x - 30\mathbf{a}_y)\sin(\omega t - 50z)$$
$$= \underline{(-3.32\mathbf{a}_x + 3.98\mathbf{a}_y)\sin(1.5 \times 10^{10}t - 50z)\ \text{A/m}^2}$$

c) Find the total magnetic flux Φ passing through the rectangle defined by $0 < x < 1$, $y = 0$, $0 < z < 1$, at $t = 0$: In free space, the magnetic field of the uniform plane wave can be easily found using the intrinsic impedance:

$$\mathbf{H}(z,t) = \left(\frac{25}{\eta_0}\mathbf{a}_y + \frac{30}{\eta_0}\mathbf{a}_x\right)\cos(\omega t - 50z)\ \text{A/m}$$

Then $\mathbf{B}(z,t) = \mu_0\mathbf{H}(z,t) = (1/c)(25\mathbf{a}_y + 30\mathbf{a}_x)\cos(\omega t - 50z)$ Wb/m^2, where $\mu_0/\eta_0 = \sqrt{\mu_0\epsilon_0} = 1/c$. The flux at $t = 0$ is now

$$\Phi = \int_0^1\int_0^1 \mathbf{B}\cdot\mathbf{a}_y\,dx\,dz = \int_0^1 \frac{25}{c}\cos(50z)\,dz = \frac{25}{50(3 \times 10^8)}\sin(50) = \underline{-0.44\ \text{nWb}}$$

11.7. The phasor magnetic field intensity for a 400-MHz uniform plane wave propagating in a certain lossless material is $(2\mathbf{a}_y - j5\mathbf{a}_z)e^{-j25x}$ A/m. Knowing that the maximum amplitude of $\mathbf{E}$ is 1500 V/m, find β, η, λ, v_p, ϵ_R, μ_R, and $\mathbf{H}(x,y,z,t)$: First, from the phasor expression, we identify $\beta = \underline{25\ \text{m}^{-1}}$ from the argument of the exponential function. Next, we evaluate $H_0 = |\mathbf{H}| = \sqrt{\mathbf{H}\cdot\mathbf{H}^*} = \sqrt{2^2 + 5^2} = \sqrt{29}$. Then $\eta = E_0/H_0 = 1500/\sqrt{29} = \underline{278.5\ \Omega}$. Then $\lambda = 2\pi/\beta = 2\pi/25 = .25\ \text{m} = \underline{25\ \text{cm}}$. Next,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 400 \times 10^6}{25} = \underline{1.01 \times 10^8\ \text{m/s}}$$

Now we note that

$$\eta = 278.5 = 377\sqrt{\frac{\mu_R}{\epsilon_R}} \quad\Rightarrow\quad \frac{\mu_R}{\epsilon_R} = 0.546$$

And

$$v_p = 1.01 \times 10^8 = \frac{c}{\sqrt{\mu_R\epsilon_R}} \quad\Rightarrow\quad \mu_R\epsilon_R = 8.79$$

We solve the above two equations simultaneously to find $\epsilon_R = \underline{4.01}$ and $\mu_R = \underline{2.19}$. Finally,

$$\mathbf{H}(x,y,z,t) = Re\left\{(2\mathbf{a}_y - j5\mathbf{a}_z)e^{-j25x}e^{j\omega t}\right\}$$
$$= 2\cos(2\pi \times 400 \times 10^6 t - 25x)\mathbf{a}_y + 5\sin(2\pi \times 400 \times 10^6 t - 25x)\mathbf{a}_z$$
$$= \underline{2\cos(8\pi \times 10^8 t - 25x)\mathbf{a}_y + 5\sin(8\pi \times 10^8 t - 25x)\mathbf{a}_z\ \text{A/m}}$$

11.8. Let the fields, $\mathbf{E}(z,t) = 1800\cos(10^7\pi t - \beta z)\mathbf{a}_x$ V/m and $\mathbf{H}(z,t) = 3.8\cos(10^7\pi t - \beta z)\mathbf{a}_y$ A/m, represent a uniform plane wave propagating at a velocity of 1.4×10^8 m/s in a perfect dielectric. Find:

a) $\beta = \omega/v = (10^7\pi)/(1.4 \times 10^8) = \underline{0.224\,\mathrm{m}^{-1}}$.

b) $\lambda = 2\pi/\beta = 2\pi/.224 = \underline{28.0\,\mathrm{m}}$.

c) $\eta = |\mathbf{E}|/|\mathbf{H}| = 1800/3.8 = \underline{474\,\Omega}$.

d) μ_R: Have two equations in the two unknowns, μ_R and ϵ_R: $\eta = \eta_0\sqrt{\mu_R/\epsilon_R}$ and $\beta = \omega\sqrt{\mu_R\epsilon_R}/c$. Eliminate ϵ_R to find

$$\mu_R = \left[\frac{\beta c\eta}{\omega\eta_0}\right]^2 = \left[\frac{(.224)(3\times10^8)(474)}{(10^7\pi)(377)}\right]^2 = \underline{2.69}$$

e) $\epsilon_R = \mu_R(\eta_0/\eta)^2 = (2.69)(377/474)^2 = \underline{1.70}$.

11.9. A certain lossless material has $\mu_R = 4$ and $\epsilon_R = 9$. A 10-MHz uniform plane wave is propagating in the $\mathbf{a}_y$ direction with $E_{x0} = 400$ V/m and $E_{y0} = E_{z0} = 0$ at $P(0.6, 0.6, 0.6)$ at $t = 60$ ns.

a) Find β, λ, v_p, and η: For a uniform plane wave,

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\sqrt{\mu_R\epsilon_R} = \frac{2\pi\times10^7}{3\times10^8}\sqrt{(4)(9)} = \underline{0.4\pi\ \mathrm{rad/m}}$$

Then $\lambda = (2\pi)/\beta = (2\pi)/(0.4\pi) = \underline{5\,\mathrm{m}}$. Next,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi\times10^7}{4\pi\times10^{-1}} = \underline{5\times10^7\ \mathrm{m/s}}$$

Finally,

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0\sqrt{\frac{\mu_R}{\epsilon_R}} = 377\sqrt{\frac{4}{9}} = \underline{251\ \Omega}$$

b) Find $E(t)$ (at P): We are given the amplitude at $t = 60$ ns and at $y = 0.6$ m. Let the maximum amplitude be E_{max}, so that in general, $E_x = E_{max}\cos(\omega t - \beta y)$. At the given position and time,

$$\begin{aligned}E_x &= 400 = E_{max}\cos[(2\pi\times10^7)(60\times10^{-9}) - (4\pi\times10^{-1})(0.6)] = E_{max}\cos(0.96\pi)\\ &= -0.99E_{max}\end{aligned}$$

So $E_{max} = (400)/(-0.99) = -403$ V/m. Thus at P, $E(t) = \underline{-403\cos(2\pi\times10^7 t)\ \mathrm{V/m}}$.

c) Find $H(t)$: First, we note that if E at a given instant points in the negative x direction, while the wave propagates in the forward y direction, then H at that same position and time must point in the positive z direction. Since we have a lossless homogeneous medium, η is real, and we are allowed to write $H(t) = E(t)/\eta$, where η is treated as negative and real. Thus

$$H(t) = H_z(t) = \frac{E_x(t)}{\eta} = \frac{-403}{-251}\cos(2\pi\times10^{-7}t) = \underline{1.61\cos(2\pi\times10^{-7}t)\ \mathrm{A/m}}$$

11.10. Given a 20MHz uniform plane wave with $\mathbf{H}_s = (6\mathbf{a}_x - j2\mathbf{a}_y)e^{-jz}$ A/m, assume propagation in a lossless medium characterized by $\epsilon_R = 5$ and an unknown μ_R.

a) Find λ, v_p, μ_R, and η: First, $\beta = 1$, so $\lambda = 2\pi/\beta = \underline{2\pi \text{ m}}$. Next, $v_p = \omega/\beta = 2\pi \times 20 \times 10^6 = \underline{4\pi \times 10^7 \text{ m/s}}$. Then, $\mu_R = (\beta^2 c^2)/(\omega^2 \epsilon_R) = (3 \times 10^8)^2/(4\pi \times 10^7)^2(5) = \underline{1.14}$. Finally, $\eta = \eta_0\sqrt{\mu_R/\epsilon_R} = 377\sqrt{1.14/5} = \underline{180}$.

b) Determine $\mathbf{E}$ at the origin at $t = 20$ns: We use the relation $|\mathbf{E}| = \eta|\mathbf{H}|$ and note that for positive z propagation, a positive x component of $\mathbf{H}$ is coupled to a negative y component of $\mathbf{E}$, and a negative y component of $\mathbf{H}$ is coupled to a negative x component of $\mathbf{E}$. We obtain $\mathbf{E}_s = -\eta(6\mathbf{a}_y + j2\mathbf{a}_x)e^{-jz}$. Then $\mathbf{E}(z,t) = \text{Re}\left\{\mathbf{E}_s e^{j\omega t}\right\} = -6\eta\cos(\omega t - z)\mathbf{a}_y + 2\eta\sin(\omega t - z)\mathbf{a}_x = 360\sin(\omega t - z)\mathbf{a}_x - 1080\cos(\omega t - z)\mathbf{a}_y$. With $\omega = 4\pi \times 10^7 \text{ sec}^{-1}$, $t = 2 \times 10^{-8}$ s, and $z = 0$, $\mathbf{E}$ evaluates as $\mathbf{E}(0, 20\text{ns}) = 360(0.588)\mathbf{a}_x - 1080(-0.809)\mathbf{a}_y = \underline{212\mathbf{a}_x + 874\mathbf{a}_y \text{ V/m}}$.

11.11. A 2-GHz uniform plane wave has an amplitude of $E_{y0} = 1.4$ kV/m at $(0, 0, 0, t = 0)$ and is propagating in the $\mathbf{a}_z$ direction in a medium where $\epsilon'' = 1.6 \times 10^{-11}$ F/m, $\epsilon' = 3.0 \times 10^{-11}$ F/m, and $\mu = 2.5\,\mu$H/m. Find:

a) E_y at $P(0, 0, 1.8\text{cm})$ at 0.2 ns: To begin, we have the ratio, $\epsilon''/\epsilon' = 1.6/3.0 = 0.533$. So

$$\alpha = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right]^{1/2}$$

$$= (2\pi \times 2 \times 10^9)\sqrt{\frac{(2.5 \times 10^{-6})(3.0 \times 10^{-11})}{2}}\left[\sqrt{1 + (.533)^2} - 1\right]^{1/2} = 28.1 \,\text{Np/m}$$

Then

$$\beta = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right]^{1/2} = 112 \,\text{rad/m}$$

Thus in general,

$$E_y(z,t) = 1.4e^{-28.1z}\cos(4\pi \times 10^9 t - 112z) \text{ kV/m}$$

Evaluating this at $t = 0.2$ ns and $z = 1.8$ cm, find

$$E_y(1.8\,\text{cm}, 0.2\,\text{ns}) = \underline{0.74 \text{ kV/m}}$$

b) H_x at P at 0.2 ns: We use the phasor relation, $H_{xs} = -E_{ys}/\eta$ where

$$\eta = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1 - j(\epsilon''/\epsilon')}} = \sqrt{\frac{2.5 \times 10^{-6}}{3.0 \times 10^{-11}}}\frac{1}{\sqrt{1 - j(.533)}} = 263 + j65.7 = 271\angle\,14^\circ\ \Omega$$

So now

$$H_{xs} = -\frac{E_{ys}}{\eta} = -\frac{(1.4 \times 10^3)e^{-28.1z}e^{-j112z}}{271e^{j14^\circ}} = -5.16e^{-28.1z}e^{-j112z}e^{-j14^\circ} \text{ A/m}$$

Then

$$H_x(z,t) = -5.16e^{-28.1z}\cos(4\pi \times 10^{-9}t - 112z - 14^\circ)$$

This, when evaluated at $t = 0.2$ ns and $z = 1.8$ cm, yields

$$H_x(1.8\,\text{cm}, 0.2\,\text{ns}) = \underline{-3.0\,\text{A/m}}$$

11.12. The plane wave $\mathbf{E}_s = 300e^{-jkx}\mathbf{a}_y$ V/m is propagating in a material for which $\mu = 2.25\,\mu$H/m, $\epsilon' = 9$ pF/m, and $\epsilon'' = 7.8$ pF/m. If $\omega = 64$ Mrad/s, find:

a) α: We use the general formula, Eq. (35):

$$\alpha = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right]^{1/2}$$
$$= (64\times 10^6)\sqrt{\frac{(2.25\times 10^{-6})(9\times 10^{-12})}{2}}\left[\sqrt{1+(.867)^2} - 1\right]^{1/2} = \underline{0.116 \text{ Np/m}}$$

b) β: Using (36), we write

$$\beta = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right]^{1/2} = \underline{.311 \text{ rad/m}}$$

c) $v_p = \omega/\beta = (64\times 10^6)/(.311) = \underline{2.06\times 10^8 \text{ m/s}}$.

d) $\lambda = 2\pi/\beta = 2\pi/(.311) = \underline{20.2 \text{ m}}$.

e) η: Using (39):

$$\eta = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1-j(\epsilon''/\epsilon')}} = \sqrt{\frac{2.25\times 10^{-6}}{9\times 10^{-12}}}\frac{1}{\sqrt{1-j(.867)}} = 407 + j152 = \underline{434.5e^{j.36}\ \Omega}$$

f) $\mathbf{H}_s$: With $\mathbf{E}_s$ in the positive y direction (at a given time) and propagating in the positive x direction, we would have a positive z component of $\mathbf{H}_s$, at the same time. We write (with $jk = \alpha + j\beta$):

$$\mathbf{H}_s = \frac{E_s}{\eta}\mathbf{a}_z = \frac{300}{434.5e^{j.36}}e^{-jkx}\mathbf{a}_z = 0.69e^{-\alpha x}e^{-j\beta x}e^{-j.36}\mathbf{a}_z$$
$$= \underline{0.69e^{-.116x}e^{-j.311x}e^{-j.36}\mathbf{a}_z \text{ A/m}}$$

g) $\mathbf{E}(3, 2, 4, 10\text{ns})$: The real instantaneous form of $\mathbf{E}$ will be

$$\mathbf{E}(x,y,z,t) = \text{Re}\left\{\mathbf{E}_s e^{j\omega t}\right\} = 300e^{-\alpha x}\cos(\omega t - \beta x)\mathbf{a}_y$$

Therefore

$$\mathbf{E}(3, 2, 4, 10\text{ns}) = 300e^{-.116(3)}\cos[(64\times 10^6)(10^{-8}) - .311(3)]\mathbf{a}_y = \underline{203 \text{ V/m}}$$

11.13. Let $jk = 0.2 + j1.5\,\text{m}^{-1}$ and $\eta = 450 + j60\ \Omega$ for a uniform plane wave propagating in the $\mathbf{a}_z$ direction. If $\omega = 300$ Mrad/s, find μ, ϵ', and ϵ'': We begin with

$$\eta = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1-j(\epsilon''/\epsilon')}} = 450 + j60$$

and

$$jk = j\omega\sqrt{\mu\epsilon'}\sqrt{1-j(\epsilon''/\epsilon')} = 0.2 + j1.5$$

11.13. (continued) Then

$$\eta\eta^* = \frac{\mu}{\epsilon'}\frac{1}{\sqrt{1+(\epsilon''/\epsilon')^2}} = (450+j60)(450-j60) = 2.06\times 10^5 \qquad (1)$$

and

$$(jk)(jk)^* = \omega^2\mu\epsilon'\sqrt{1+(\epsilon''/\epsilon')^2} = (0.2+j1.5)(0.2-j1.5) = 2.29 \qquad (2)$$

Taking the ratio of (2) to (1),

$$\frac{(jk)(jk)^*}{\eta\eta^*} = \omega^2(\epsilon')^2\left(1+(\epsilon''/\epsilon')^2\right) = \frac{2.29}{2.06\times 10^5} = 1.11\times 10^{-5}$$

Then with $\omega = 3\times 10^8$,

$$(\epsilon')^2 = \frac{1.11\times 10^{-5}}{(3\times 10^8)^2\left(1+(\epsilon''/\epsilon')^2\right)} = \frac{1.23\times 10^{-22}}{\left(1+(\epsilon''/\epsilon')^2\right)} \qquad (3)$$

Now, we use Eqs. (35) and (36). Squaring these and taking their ratio gives

$$\frac{\alpha^2}{\beta^2} = \frac{\sqrt{1+(\epsilon''/\epsilon')^2}}{\sqrt{1+(\epsilon''/\epsilon')^2}} = \frac{(0.2)^2}{(1.5)^2}$$

We solve this to find $\epsilon''/\epsilon' = 0.271$. Substituting this result into (3) gives $\epsilon' = 1.07\times 10^{-11}$ F/m. Since $\epsilon''/\epsilon' = 0.271$, we then find $\epsilon'' = 2.90\times 10^{-12}$ F/m. Finally, using these results in either (1) or (2) we find $\mu = 2.28\times 10^{-6}$ H/m. Summary: $\mu = \underline{2.28\times 10^{-6}\ \text{H/m}}$,
$\epsilon' = \underline{1.07\times 10^{-11}\ \text{F/m}}$, and $\epsilon'' = \underline{2.90\times 10^{-12}\ \text{F/m}}$.

11.14. A certain nonmagnetic material has the material constants $\epsilon'_R = 2$ and $\epsilon''/\epsilon' = 4\times 10^{-4}$ at $\omega = 1.5$ Grad/s. Find the distance a uniform plane wave can propagate through the material before:

a) it is attenuated by 1 Np: First, $\epsilon'' = (4\times 10^4)(2)(8.854\times 10^{-12}) = 7.1\times 10^{-15}$ F/m. Then, since $\epsilon''/\epsilon' << 1$, we use the approximate form for α, given by Eq. (51) (written in terms of ϵ''):

$$\alpha \doteq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{(1.5\times 10^9)(7.1\times 10^{-15})}{2}\frac{377}{\sqrt{2}} = 1.42\times 10^{-3}\ \text{Np/m}$$

The required distance is now $z_1 = (1.42\times 10^{-3})^{-1} = \underline{706\ \text{m}}$

b) the power level is reduced by one-half: The governing relation is $e^{-2\alpha z_{1/2}} = 1/2$, or $z_{1/2} = \ln 2/2\alpha = \ln 2/2(1.42\times 10^-3) = \underline{244\ \text{m}}$.

c) the phase shifts 360°: This distance is defined as one wavelength, where $\lambda = 2\pi/\beta$ $= (2\pi c)/(\omega\sqrt{\epsilon'_R}) = [2\pi(3\times 10^8)]/[(1.5\times 10^9)\sqrt{2}] = \underline{0.89\ \text{m}}$.

11.15. A 10 GHz radar signal may be represented as a uniform plane wave in a sufficiently small region. Calculate the wavelength in centimeters and the attenuation in nepers per meter if the wave is propagating in a non-magnetic material for which

a) $\epsilon'_R = 1$ and $\epsilon''_R = 0$: In a non-magnetic material, we would have:

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon'_R}{2}}\left[\sqrt{1+\left(\frac{\epsilon''_R}{\epsilon'_R}\right)^2} - 1\right]^{1/2}$$

11.15. (continued) and

$$\beta = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon'_R}{2}}\left[\sqrt{1+\left(\frac{\epsilon''_R}{\epsilon'_R}\right)^2}+1\right]^{1/2}$$

With the given values of ϵ'_R and ϵ''_R, it is clear that $\beta = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$, and so $\lambda = 2\pi/\beta = 2\pi c/\omega = 3\times 10^{10}/10^{10} = \underline{3\text{ cm}}$. It is also clear that $\alpha = \underline{0}$.

b) $\epsilon'_R = 1.04$ and $\epsilon''_R = 9.00\times 10^{-4}$: In this case $\epsilon''_R/\epsilon'_R << 1$, and so $\beta \doteq \omega\sqrt{\epsilon'_R}/c = 2.13\text{ cm}^{-1}$. Thus $\lambda = 2\pi/\beta = \underline{2.95\text{ cm}}$. Then

$$\alpha \doteq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega\epsilon''_R}{2}\frac{\sqrt{\mu_0\epsilon_0}}{\sqrt{\epsilon'_R}} = \frac{\omega}{2c}\frac{\epsilon''_R}{\sqrt{\epsilon'_R}} = \frac{2\pi\times 10^{10}}{2\times 3\times 10^8}\frac{(9.00\times 10^{-4})}{\sqrt{1.04}}$$
$$= \underline{9.24\times 10^{-2}\text{ Np/m}}$$

c) $\epsilon'_R = 2.5$ and $\epsilon''_R = 7.2$: Using the above formulas, we obtain

$$\beta = \frac{2\pi\times 10^{10}\sqrt{2.5}}{(3\times 10^{10})\sqrt{2}}\left[\sqrt{1+\left(\frac{7.2}{2.5}\right)^2}+1\right]^{1/2} = 4.71\text{ cm}^{-1}$$

and so $\lambda = 2\pi/\beta = \underline{1.33\text{ cm}}$. Then

$$\alpha = \frac{2\pi\times 10^{10}\sqrt{2.5}}{(3\times 10^{8})\sqrt{2}}\left[\sqrt{1+\left(\frac{7.2}{2.5}\right)^2}-1\right]^{1/2} = \underline{335\text{ Np/m}}$$

11.16. The power factor of a capacitor is defined as the cosine of the impedance phase angle, and its Q is ωCR, where R is the parallel resistance. Assume an idealized parallel plate capacitor having a dielecric characterized by σ, ϵ', and μ_R. Find both the power factor and Q in terms of the loss tangent: First, the impedance will be:

$$Z = \frac{R\left(\frac{1}{j\omega C}\right)}{R+\left(\frac{1}{j\omega C}\right)} = R\frac{1-jR\omega C}{1+(R\omega C)^2} = R\frac{1-jQ}{1+Q^2}$$

Now $R = d/(\sigma A)$ and $C = \epsilon' A/d$, and so $Q = \omega\epsilon'/\sigma = \underline{1/l.t.}$ Then the power factor is P.F $= \cos[\tan^{-1}(-Q)] = \underline{1/\sqrt{1+Q^2}}$.

11.17. Let $\eta = 250 + j30\,\Omega$ and $jk = 0.2 + j2\,\text{m}^{-1}$ for a uniform plane wave propagating in the $\mathbf{a}_z$ direction in a dielectric having some finite conductivity. If $|E_s| = 400$ V/m at $z = 0$, find:

a) $\mathbf{P}_{z,av}$ at $z = 0$ and $z = 60$ cm: Assume x-polarization for the electric field. Then

$$\mathbf{P}_{z,av} = \frac{1}{2}\text{Re}\left\{\mathbf{E}_s \times \mathbf{H}_s^*\right\} = \frac{1}{2}\text{Re}\left\{400e^{-\alpha z}e^{-j\beta z}\mathbf{a}_x \times \frac{400}{\eta^*}e^{-\alpha z}e^{j\beta z}\mathbf{a}_y\right\}$$
$$= \frac{1}{2}(400)^2 e^{-2\alpha z}\text{Re}\left\{\frac{1}{\eta^*}\right\}\mathbf{a}_z = 8.0 \times 10^4 e^{-2(0.2)z}\text{Re}\left\{\frac{1}{250 - j30}\right\}\mathbf{a}_z$$
$$= 315\, e^{-2(0.2)z}\,\mathbf{a}_z \text{ W/m}^2$$

Evaluating at $z = 0$, obtain $\mathbf{P}_{z,av}(z = 0) = \underline{315\,\mathbf{a}_z \text{ W/m}^2}$,
and at $z = 60$ cm, $\mathbf{P}_{z,av}(z = 0.6) = 315e^{-2(0.2)(0.6)}\mathbf{a}_z = \underline{248\,\mathbf{a}_z \text{ W/m}^2}$.

b) the average ohmic power dissipation in watts per cubic meter at $z = 60$ cm: At this point a flaw becomes evident in the problem statement, since solving this part in two different ways gives results that are not the same. I will demonstrate: In the first method, we use Poynting's theorem in point form (first equation at the top of p. 366), which we modify for the case of time-average fields to read:

$$-\nabla \cdot \mathbf{P}_{z,av} = < \mathbf{J} \cdot \mathbf{E} >$$

where the right hand side is the average power dissipation per volume. Note that the additional right-hand-side terms in Poynting's theorem that describe changes in energy stored in the fields will both be zero in steady state. We apply our equation to the result of part a:

$$< \mathbf{J} \cdot \mathbf{E} > = -\nabla \cdot \mathbf{P}_{z,av} = -\frac{d}{dz}315\, e^{-2(0.2)z} = (0.4)(315)e^{-2(0.2)z} = 126e^{-0.4z} \text{ W/m}^3$$

At $z = 60$ cm, this becomes $< \mathbf{J} \cdot \mathbf{E} > = 99.1 \text{ W/m}^3$. In the second method, we solve for the conductivity and evaluate $< \mathbf{J} \cdot \mathbf{E} > = \sigma < E^2 >$. We use

$$jk = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j(\epsilon''/\epsilon')}$$

and

$$\eta = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1 - j(\epsilon''/\epsilon')}}$$

We take the ratio,

$$\frac{jk}{\eta} = j\omega\epsilon'\left[1 - j\left(\frac{\epsilon''}{\epsilon'}\right)\right] = j\omega\epsilon' + \omega\epsilon''$$

Identifying $\sigma = \omega\epsilon''$, we find

$$\sigma = \text{Re}\left\{\frac{jk}{\eta}\right\} = \text{Re}\left\{\frac{0.2 + j2}{250 + j30}\right\} = 1.74 \times 10^{-3} \text{ S/m}$$

Now we find the dissipated power per volume:

$$\sigma < E^2 > = 1.74 \times 10^{-3}\left(\frac{1}{2}\right)\left(400e^{-0.2z}\right)^2$$

11.17b. (continued) At $z = 60$ cm, this evaluates as 109 W/m^3. One can show that consistency between the two methods requires that

$$\text{Re}\left\{\frac{1}{\eta^*}\right\} = \frac{\sigma}{2\alpha}$$

This relation does not hold using the numbers as given in the problem statement and the value of σ found above. Note that in Problem 11.13, where all values are worked out, the relation does hold and consistent results are obtained using both methods.

11.18a. Find $P(\mathbf{r}, t)$ if $\mathbf{E}_s = 400e^{-j2x}\mathbf{a}_y$ V/m in free space: A positive y component of $\mathbf{E}$ requires a positive z component of $\mathbf{H}$ for propagation in the forward x direction. Thus $\mathbf{H}_s = (400/\eta_0)e^{-j2x}\mathbf{a}_z = 1.06e^{-j2x}\mathbf{a}_z$ A/m. In real form, the field are $\mathbf{E}(x,t) = 400\cos(\omega t - 2x)\mathbf{a}_y$ and $\mathbf{H}(x,t) = 1.06\cos(\omega t - 2x)\mathbf{a}_z$. Now $P(\mathbf{r}, t) = P(x, t) = \mathbf{E}(x,t) \times \mathbf{H}(x,t) = \underline{424.4\cos^2(\omega t - 2x)\mathbf{a}_x \text{ W/m}^2}$.

b) Find P at $t = 0$ for $\mathbf{r} = (a, 5, 10)$, where a = 0,1,2, and 3: At $t = 0$, we find from part a, $P(a, 0) = 424.4\cos^2(2a)$, which leads to the values (in W/m^2): $\underline{424.4 \text{ at } a = 0}$, $\underline{73.5 \text{ at } a = 1}$, $\underline{181.3 \text{ at } a = 2}$, and $\underline{391.3 \text{ at } a = 3}$.

c) Find P at the origin for $T = 0$, $0.2T$, $0.4T$, and $0.6T$, where T is the oscillation period. At the origin, we have $P(0, t) = 424.4\cos^2(\omega t) = 424.4\cos^2(2\pi t/T)$. Using this, we obtain the following values (in W/m^2): $\underline{424.4 \text{ at } t = 0}$, $\underline{42.4 \text{ at } t = 0.2T}$, $\underline{277.8 \text{ at } t = 0.4T}$, and $\underline{277.8 \text{ at } t = 0.6T}$.

11.19. Perfectly-conducting cylinders with radii of 8 mm and 20 mm are coaxial. The region between the cylinders is filled with a perfect dielectric for which $\epsilon = 10^{-9}/4\pi$ F/m and $\mu_R = 1$. If $\mathbf{E}$ in this region is $(500/\rho)\cos(\omega t - 4z)\mathbf{a}_\rho$ V/m, find:

a) ω, with the help of Maxwell's equations in cylindrical coordinates: We use the two curl equations, beginning with $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$, where in this case,

$$\nabla \times \mathbf{E} = \frac{\partial E_\rho}{\partial z}\mathbf{a}_\phi = \frac{2000}{\rho}\sin(\omega t - 4z)\mathbf{a}_\phi = -\frac{\partial B_\phi}{\partial t}\mathbf{a}_\phi$$

So

$$B_\phi = \int \frac{2000}{\rho}\sin(\omega t - 4z)dt = \frac{2000}{\omega\rho}\cos(\omega t - 4z) \text{ T}$$

Then

$$H_\phi = \frac{B_\phi}{\mu_0} = \frac{2000}{(4\pi \times 10^{-7})\omega\rho}\cos(\omega t - 4z) \text{ A/m}$$

We next use $\nabla \times \mathbf{H} = \partial\mathbf{D}/\partial t$, where in this case

$$\nabla \times \mathbf{H} = -\frac{\partial H_\phi}{\partial z}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho}\mathbf{a}_z$$

where the second term on the right hand side becomes zero when substituting our H_ϕ. So

$$\nabla \times \mathbf{H} = -\frac{\partial H_\phi}{\partial z}\mathbf{a}_\rho = -\frac{8000}{(4\pi \times 10^{-7})\omega\rho}\sin(\omega t - 4z)\mathbf{a}_\rho = \frac{\partial D_\rho}{\partial t}\mathbf{a}_\rho$$

And

$$D_\rho = \int -\frac{8000}{(4\pi \times 10^{-7})\omega\rho}\sin(\omega t - 4z)dt = \frac{8000}{(4\pi \times 10^{-7})\omega^2\rho}\cos(\omega t - 4z) \text{ C/m}^2$$

11.19a. (continued) Finally, using the given ϵ,

$$E_\rho = \frac{D_\rho}{\epsilon} = \frac{8000}{(10^{-16})\omega^2\rho}\cos(\omega t - 4z) \text{ V/m}$$

This must be the same as the given field, so we require

$$\frac{8000}{(10^{-16})\omega^2\rho} = \frac{500}{\rho} \quad\Rightarrow\quad \omega = \underline{4 \times 10^8 \text{ rad/s}}$$

b) $\mathbf{H}(\rho, z, t)$: From part a, we have

$$\mathbf{H}(\rho, z, t) = \frac{2000}{(4\pi \times 10^{-7})\omega\rho}\cos(\omega t - 4z)\mathbf{a}_\phi = \underline{\frac{4.0}{\rho}\cos(4 \times 10^8 t - 4z)\mathbf{a}_\phi \text{ A/m}}$$

c) $\mathbf{P}(\rho, \phi, z)$: This will be

$$\mathbf{P}(\rho, \phi, z) = \mathbf{E} \times \mathbf{H} = \frac{500}{\rho}\cos(4 \times 10^8 t - 4z)\mathbf{a}_\rho \times \frac{4.0}{\rho}\cos(4 \times 10^8 t - 4z)\mathbf{a}_\phi$$
$$= \underline{\frac{2.0 \times 10^{-3}}{\rho^2}\cos^2(4 \times 10^8 t - 4z)\mathbf{a}_z \text{ W/m}^2}$$

d) the average power passing through every cross-section $8 < \rho < 20$ mm, $0 < \phi < 2\pi$. Using the result of part c, we find $\mathbf{P}_{avg} = (1.0 \times 10^3)/\rho^2 \mathbf{a}_z$ W/m^2. The power through the given cross-section is now

$$\text{P} = \int_0^{2\pi}\int_{.008}^{.020} \frac{1.0 \times 10^3}{\rho^2}\,\rho\,d\rho\,d\phi = 2\pi \times 10^3 \ln\left(\frac{20}{8}\right) = \underline{5.7 \text{ kW}}$$

11.20. If $\mathbf{E}_s = (60/r)\sin\theta\, e^{-j2r}\,\mathbf{a}_\theta$ V/m, and $\mathbf{H}_s = (1/4\pi r)\sin\theta\, e^{-j2r}\,\mathbf{a}_\phi$ A/m in free space, find the average power passing outward through the surface $r = 10^6$, $0 < \theta < \pi/3$, and $0 < \phi < 2\pi$.

$$P_{avg} = \frac{1}{2}\text{Re}\left\{\mathbf{E}_s \times \mathbf{H}_s^*\right\} = \frac{15\sin^2\theta}{2\pi r^2}\,\mathbf{a}_r \text{ W/m}^2$$

Then, the requested power will be

$$\Phi = \int_0^{2\pi}\int_0^{\pi/3} \frac{15\sin^2\theta}{2\pi r^2}\,\mathbf{a}_r \cdot \mathbf{a}_r\, r^2 \sin\theta d\theta d\phi = 15\int_0^{\pi/3}\sin^3\theta\, d\theta$$
$$= 15\left(-\frac{1}{3}\cos\theta(\sin^2\theta + 2)\right)\Bigg|_0^{\pi/3} = \frac{25}{8} = \underline{3.13 \text{ W}}$$

Note that the radial distance at the surface, $r = 10^6$ m, makes no difference, since the power density dimishes as $1/r^2$.

11.21. The cylindrical shell, 1 cm < ρ < 1.2 cm, is composed of a conducting material for which $\sigma = 10^6$ S/m. The external and internal regions are non-conducting. Let $H_\phi = 2000$ A/m at $\rho = 1.2$ cm.

a) Find **H** everywhere: Use Ampere's circuital law, which states:

$$\oint \mathbf{H} \cdot d\mathbf{L} = 2\pi\rho(2000) = 2\pi(1.2 \times 10^{-2})(2000) = 48\pi \text{ A} = I_{encl}$$

Then in this case

$$\mathbf{J} = \frac{I}{Area}\mathbf{a}_z = \frac{48}{(1.44 - 1.00) \times 10^{-4}}\mathbf{a}_z = 1.09 \times 10^6\,\mathbf{a}_z \text{ A/m}^2$$

With this result we again use Ampere's circuital law to find **H** everywhere within the shell as a function of ρ (in meters):

$$H_{\phi 1}(\rho) = \frac{1}{2\pi\rho}\int_0^{2\pi}\int_{.01}^{\rho} 1.09 \times 10^6\ \rho\, d\rho\, d\phi = \underline{\frac{54.5}{\rho}(10^4\rho^2 - 1) \text{ A/m} \ \ (.01 < \rho < .012)}$$

Outside the shell, we would have

$$H_{\phi 2}(\rho) = \frac{48\pi}{2\pi\rho} = \underline{24/\rho \text{ A/m} \ \ (\rho > .012)}$$

Inside the shell ($\rho < .01$ m), $H_\phi = 0$ since there is no enclosed current.

b) Find **E** everywhere: We use

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} = \frac{1.09 \times 10^6}{10^6}\mathbf{a}_z = \underline{1.09\,\mathbf{a}_z \text{ V/m}}$$

which is valid, presumeably, outside as well as inside the shell.

c) Find **P** everywhere: Use

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} = 1.09\,\mathbf{a}_z \times \frac{54.5}{\rho}(10^4\rho^2 - 1)\,\mathbf{a}_\phi$$
$$= \underline{-\frac{59.4}{\rho}(10^4\rho^2 - 1)\,\mathbf{a}_\rho \text{ W/m}^2 \ \ (.01 < \rho < .012 \text{ m})}$$

Outside the shell,

$$\mathbf{P} = 1.09\,\mathbf{a}_z \times \frac{24}{\rho}\,\mathbf{a}_\phi = \underline{-\frac{26}{\rho}\,\mathbf{a}_\rho \text{ W/m}^2 \ \ (\rho > .012 \text{ m})}$$

11.22. The inner and outer dimensions of a copper coaxial transmission line are 2 and 7 mm, respectively. Both conductors have thicknesses much greater than δ. The dielectric is lossless and the operating frequency is 400 MHz. Calculate the resistance per meter length of the:

a) inner conductor: First

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi(4\times 10^8)(4\pi \times 10^{-7})(5.8\times 10^7)}} = 3.3\times 10^{-6}\text{m} = 3.3\mu\text{m}$$

Now, using (70) with a unit length, we find

$$R_{in} = \frac{1}{2\pi a \sigma \delta} = \frac{1}{2\pi(2\times 10^{-3})(5.8\times 10^7)(3.3\times 10^{-6})} = \underline{0.42 \text{ ohms/m}}$$

b) outer conductor: Again, (70) applies but with a different conductor radius. Thus

$$R_{out} = \frac{a}{b} R_{in} = \frac{2}{7}(0.42) = \underline{0.12 \text{ ohms/m}}$$

c) transmission line: Since the two resistances found above are in series, the line resistance is their sum, or $R = R_{in} + R_{out} = \underline{0.54 \text{ ohms/m}}$.

11.23. A hollow tubular conductor is constructed from a type of brass having a conductivity of 1.2×10^7 S/m. The inner and outer radii are 9 mm and 10 mm respectively. Calculate the resistance per meter length at a frequency of

a) dc: In this case the current density is uniform over the entire tube cross-section. We write:

$$R(\text{dc}) = \frac{L}{\sigma A} = \frac{1}{(1.2\times 10^7)\pi(.01^2 - .009^2)} = \underline{1.4\times 10^{-3}\ \Omega/\text{m}}$$

b) 20 MHz: Now the skin effect will limit the effective cross-section. At 20 MHz, the skin depth is

$$\delta(20\text{MHz}) = [\pi f \mu_0 \sigma]^{-1/2} = [\pi(20\times 10^6)(4\pi \times 10^{-7})(1.2\times 10^7)]^{-1/2} = 3.25\times 10^{-5}\text{ m}$$

This is much less than the outer radius of the tube. Therefore we can approximate the resistance using the formula:

$$R(20\text{MHz}) = \frac{L}{\sigma A} = \frac{1}{2\pi b \delta} = \frac{1}{(1.2\times 10^7)(2\pi(.01))(3.25\times 10^{-5})} = \underline{4.1\times 10^{-2}\ \Omega/\text{m}}$$

c) 2 GHz: Using the same formula as in part b, we find the skin depth at 2 GHz to be $\delta = 3.25\times 10^{-6}$ m. The resistance (using the other formula) is $R(2\text{GHz}) = \underline{4.1\times 10^{-1}\ \Omega/\text{m}}$.

11.24a. Most microwave ovens operate at 2.45 GHz. Assume that $\sigma = 1.2 \times 10^6$ S/m and $\mu_R = 500$ for the stainless steel interior, and find the depth of penetration:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi(2.45 \times 10^9)(4\pi \times 10^{-7})(1.2 \times 10^6)}} = 9.28 \times 10^{-6}\text{m} = 9.28\mu\text{m}$$

b) Let $E_s = 50\angle\, 0^\circ$ V/m at the surface of the conductor, and plot a curve of the amplitude of E_s vs. the angle of E_s as the field propagates into the stainless steel: Since the conductivity is high, we use (62) to write $\alpha \doteq \beta \doteq \sqrt{\pi f \mu \sigma} = 1/\delta$. So, assuming that the direction into the conductor is z, the depth-dependent field is written as

$$E_s(z) = 50e^{-\alpha z}e^{-j\beta z} = 50e^{-z/\delta}e^{-jz/\delta} = \underbrace{50\exp(-z/9.28)}_{\text{amplitude}}\exp(-j\underbrace{z/9.28}_{\text{angle}})$$

where z is in microns. Therefore, the plot of amplitude versus angle is simply a plot of e^{-x} versus x, where $x = z/9.28$; the starting amplitude is 50 and the $1/e$ amplitude (at $z = 9.28\ \mu$m) is 18.4.

11.25. A good conductor is planar in form and carries a uniform plane wave that has a wavelength of 0.3 mm and a velocity of 3×10^5 m/s. Assuming the conductor is non-magnetic, determine the frequency and the conductivity: First, we use

$$f = \frac{v}{\lambda} = \frac{3 \times 10^5}{3 \times 10^{-4}} = 10^9\ \text{Hz} = \underline{1\ \text{GHz}}$$

Next, for a good conductor,

$$\delta = \frac{\lambda}{2\pi} = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad \Rightarrow \quad \sigma = \frac{4\pi}{\lambda^2 f \mu} = \frac{4\pi}{(9 \times 10^{-8})(10^9)(4\pi \times 10^{-7})} = \underline{1.1 \times 10^5\ \text{S/m}}$$

11.26. The dimensions of a certain coaxial transmission line are $a = 0.8$mm and $b = 4$mm. The outer conductor thickness is 0.6mm, and all conductors have $\sigma = 1.6 \times 10^7$ S/m.

a) Find R, the resistance per unit length, at an operating frequency of 2.4 GHz: First

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi(2.4 \times 10^8)(4\pi \times 10^{-7})(1.6 \times 10^7)}} = 2.57 \times 10^{-6}\text{m} = 2.57\mu\text{m}$$

Then, using (70) with a unit length, we find

$$R_{in} = \frac{1}{2\pi a \sigma \delta} = \frac{1}{2\pi(0.8 \times 10^{-3})(1.6 \times 10^7)(2.57 \times 10^{-6})} = 4.84\ \text{ohms/m}$$

The outer conductor resistance is then found from the inner through

$$R_{out} = \frac{a}{b}R_{in} = \frac{0.8}{4}(4.84) = 0.97\ \text{ohms/m}$$

The net resistance per length is then the sum, $R = R_{in} + R_{out} = \underline{5.81\ \text{ohms/m}}$.

11.26b. Use information from Secs. 5.10 and 9.10 to find C and L, the capacitance and inductance per unit length, respectively. The coax is air-filled. From those sections, we find (in free space)

$$C = \frac{2\pi\epsilon_0}{\ln(b/a)} = \frac{2\pi(8.854\times10^{-12})}{\ln(4/.8)} = \underline{3.46\times10^{-11}\ \text{F/m}}$$

$$L = \frac{\mu_0}{2\pi}\ln(b/a) = \frac{4\pi\times10^{-7}}{2\pi}\ln(4/.8) = \underline{3.22\times10^{-7}\ \text{H/m}}$$

c) Find α and β if $\alpha + j\beta = \sqrt{j\omega C(R+j\omega L)}$: Taking real and imaginary parts of the given expression, we find

$$\alpha = \text{Re}\left\{\sqrt{j\omega C(R+j\omega L)}\right\} = \frac{\omega\sqrt{LC}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{R}{\omega L}\right)^2} - 1\right]^{1/2}$$

and

$$\beta = \text{Im}\left\{\sqrt{j\omega C(R+j\omega L)}\right\} = \frac{\omega\sqrt{LC}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{R}{\omega L}\right)^2} + 1\right]^{1/2}$$

These can be found by writing out $\alpha = \text{Re}\left\{\sqrt{j\omega C(R+j\omega L)}\right\} = (1/2)\sqrt{j\omega C(R+j\omega L)} + c.c.$, where $c.c$ denotes the complex conjugate. The result is squared, terms collected, and the square root taken. Now, using the values of R, C, and L found in parts a and b, we find $\alpha = \underline{3.0\times10^{-2}\ \text{Np/m}}$ and $\beta = \underline{50.3\ \text{rad/m}}$.

11.27. The planar surface at $z = 0$ is a brass-Teflon interface. Use data available in Appendix C to evaluate the following ratios for a uniform plane wave having $\omega = 4\times10^{10}$ rad/s:

a) $\alpha_{\text{Tef}}/\alpha_{\text{brass}}$: From the appendix we find $\epsilon''/\epsilon' = .0003$ for Teflon, making the material a good dielectric. Also, for Teflon, $\epsilon'_R = 2.1$. For brass, we find $\sigma = 1.5\times10^7$ S/m, making brass a good conductor at the stated frequency. For a good dielectric (Teflon) we use the approximations:

$$\alpha \doteq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon'}} = \left(\frac{\epsilon''}{\epsilon'}\right)\left(\frac{1}{2}\right)\omega\sqrt{\mu\epsilon'} = \frac{1}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\omega}{c}\sqrt{\epsilon'_R}$$

$$\beta \doteq \omega\sqrt{\mu\epsilon'}\left[1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)\right] \doteq \omega\sqrt{\mu\epsilon'} = \frac{\omega}{c}\sqrt{\epsilon'_R}$$

For brass (good conductor) we have

$$\alpha \doteq \beta \doteq \sqrt{\pi f\mu\sigma_{\text{brass}}} = \sqrt{\pi\left(\frac{1}{2\pi}\right)(4\times10^{10})(4\pi\times10^{-7})(1.5\times10^7)} = 6.14\times10^5\ \text{m}^{-1}$$

Now

$$\frac{\alpha_{\text{Tef}}}{\alpha_{\text{brass}}} = \frac{1/2\left(\epsilon''/\epsilon'\right)(\omega/c)\sqrt{\epsilon'_R}}{\sqrt{\pi f\mu\sigma_{\text{brass}}}} = \frac{(1/2)(.0003)(4\times10^{10}/3\times10^8)\sqrt{2.1}}{6.14\times10^5} = \underline{4.7\times10^{-8}}$$

b)

$$\frac{\lambda_{\text{Tef}}}{\lambda_{\text{brass}}} = \frac{(2\pi/\beta_{\text{Tef}})}{(2\pi/\beta_{\text{brass}})} = \frac{\beta_{\text{brass}}}{\beta_{\text{Tef}}} = \frac{c\sqrt{\pi f\mu\sigma_{\text{brass}}}}{\omega\sqrt{\epsilon'_{R\,\text{Tef}}}} = \frac{(3\times10^8)(6.14\times10^5)}{(4\times10^{10})\sqrt{2.1}} = \underline{3.2\times10^3}$$

11.27. (continued)

c)

$$\frac{v_{\text{Tef}}}{v_{\text{brass}}} = \frac{(\omega/\beta_{\text{Tef}})}{(\omega/\beta_{\text{brass}})} = \frac{\beta_{\text{brass}}}{\beta_{\text{Tef}}} = \underline{3.2 \times 10^3} \text{ as before}$$

11.28. A uniform plane wave in free space has electric field given by $\mathbf{E}_s = 10e^{-j\beta x}\mathbf{a}_z + 15e^{-j\beta x}\mathbf{a}_y$ V/m.

a) Describe the wave polarization: Since the two components have a fixed phase difference (in this case zero) with respect to time and position, the wave has linear polarization, with the field vector in the yz plane at angle $\phi = \tan^{-1}(10/15) = 33.7^\circ$ to the y axis.

b) Find $\mathbf{H}_s$: With propagation in forward x, we would have

$$\mathbf{H}_s = \frac{-10}{377}e^{-j\beta x}\mathbf{a}_y + \frac{15}{377}e^{-j\beta x}\mathbf{a}_z \text{ A/m} = \underline{-26.5e^{-j\beta x}\mathbf{a}_y + 39.8e^{-j\beta x}\mathbf{a}_z \text{ mA/m}}$$

c) determine the average power density in the wave in W/m^2: Use

$$\mathbf{P}_{avg} = \frac{1}{2}\text{Re}\left\{\mathbf{E}_s \times \mathbf{H}_s^*\right\} = \frac{1}{2}\left[\frac{(10)^2}{377}\mathbf{a}_x + \frac{(15)^2}{377}\mathbf{a}_x\right] = 0.43\mathbf{a}_x \text{ W/m}^2 \text{ or } P_{avg} = \underline{0.43 \text{ W/m}^2}$$

11.29. Consider a left-circularly polarized wave in free space that propagates in the forward z direction. The electric field is given by the appropriate form of Eq. (80).

a) Determine the magnetic field phasor, $\mathbf{H}_s$:
We begin, using (80), with $\mathbf{E}_s = E_0(\mathbf{a}_x + j\mathbf{a}_y)e^{-j\beta z}$. We find the two components of $\mathbf{H}_s$ separately, using the two components of $\mathbf{E}_s$. Specifically, the x component of $\mathbf{E}_s$ is associated with a y component of $\mathbf{H}_s$, and the y component of $\mathbf{E}_s$ is associated with a negative x component of $\mathbf{H}_s$. The result is

$$\mathbf{H}_s = \underline{\frac{E_0}{\eta_0}\left(\mathbf{a}_y - j\mathbf{a}_x\right)e^{-j\beta z}}$$

b) Determine an expression for the average power density in the wave in W/m^2 by direct application of Eq. (57): We have

$$\mathbf{P}_{z,avg} = \frac{1}{2}Re(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{1}{2}Re\left(E_0(\mathbf{a}_x + j\mathbf{a}_y)e^{-j\beta z} \times \frac{E_0}{\eta_0}(\mathbf{a}_y - j\mathbf{a}_x)e^{+j\beta z}\right)$$

$$= \underline{\frac{E_0^2}{\eta_0}\mathbf{a}_z \text{ W/m}^2} \text{ (assuming } E_0 \text{ is real)}$$

11.30. The electric field of a uniform plane wave in free space is given by $\mathbf{E}_s = 10(\mathbf{a}_y + j\mathbf{a}_z)e^{-j25x}$.

a) Determine the frequency, f: Use

$$f = \frac{\beta c}{2\pi} = \frac{(25)(3 \times 10^8)}{2\pi} = \underline{1.2 \text{ GHz}}$$

b) Find the magnetic field phasor, $\mathbf{H}_s$: With the Poynting vector in the positive x direction, a positive y component for $\mathbf{E}$ requires a positive z component for $\mathbf{H}$. Similarly, a positive z component for $\mathbf{E}$ requires a negative y component for $\mathbf{H}$. Therefore,

$$\underline{\mathbf{H}_s = \frac{10}{\eta_0}\left[\mathbf{a}_z - j\mathbf{a}_y\right]e^{-j25x}}$$

c) Describe the polarization of the wave: This is most clearly seen by first converting the given field to real instantaneous form:

$$\mathbf{E}(x,t) = \text{Re}\left\{\mathbf{E}_s e^{j\omega t}\right\} = 10\left[\cos(\omega t - 25x)\mathbf{a}_y - \sin(\omega t - 25x)\mathbf{a}_z\right]$$

At $x = 0$, this becomes,

$$\mathbf{E}(0,t) = 10\left[\cos(\omega t)\mathbf{a}_y - \sin(\omega t)\mathbf{a}_z\right]$$

With the wave traveling in the forward x direction, we recognize the polarization as <u>left circular</u>.

11.31. A linearly-polarized uniform plane wave, propagating in the forward z direction, is input to a lossless *anisotropic* material, in which the dielectric constant encountered by waves polarized along y (ϵ_{Ry}) differs from that seen by waves polarized along x (ϵ_{Rx}). Suppose $\epsilon_{Rx} = 2.15$, $\epsilon_{Ry} = 2.10$, and the wave electric field at input is polarized at 45° to the positive x and y axes. *Assume free space wavelength* λ.

a) Determine the shortest length of the material such that the wave as it emerges from the output end is circularly polarized: With the input field at 45°, the x and y components are of equal magnitude, and circular polarization will result if the phase difference between the components is $\pi/2$. Our requirement over length L is thus $\beta_x L - \beta_y L = \pi/2$, or

$$L = \frac{\pi}{2(\beta_x - \beta_y)} = \frac{\pi c}{2\omega(\sqrt{\epsilon_{Rx}} - \sqrt{\epsilon_{Ry}})}$$

With the given values, we find,

$$L = \frac{(58.3)\pi c}{2\omega} = 58.3\,\frac{\lambda}{4} = \underline{14.6\,\lambda}$$

b) Will the output wave be right- or left-circularly-polarized? With the dielectric constant greater for x-polarized waves, the x component will lag the y component in time at the output. The field can thus be written as $\mathbf{E} = E_0(\mathbf{a}_y - j\mathbf{a}_x)$, which is <u>left circular polarization</u>.

11.32. Suppose that the length of the medium of Problem 11.31 is made to be *twice* that as determined in the problem. Describe the polarization of the output wave in this case: With the length doubled, a phase shift of π radians develops between the two components. At the input, we can write the field as $\mathbf{E}_s(0) = E_0(\mathbf{a}_x + \mathbf{a}_y)$. After propagating through length L, we would have,

$$\mathbf{E}_s(L) = E_0[e^{-j\beta_x L}\mathbf{a}_x + e^{-j\beta_y L}\mathbf{a}_y] = E_0 e^{-j\beta_x L}[\mathbf{a}_x + e^{-j(\beta_y - \beta_x)L}\mathbf{a}_y]$$

where $(\beta_y - \beta_x)L = -\pi$ (since $\beta_x > \beta_y$), and so $\mathbf{E}_s(L) = E_0 e^{-j\beta_x L}[\mathbf{a}_x - \mathbf{a}_y]$. With the reversal of the y component, the wave polarization is rotated by 90°, but is still linear polarization.

11.33. Given a wave for which $\mathbf{E}_s = 15e^{-j\beta z}\mathbf{a}_x + 18e^{-j\beta z}e^{j\phi}\mathbf{a}_y$ V/m, propagating in a medium characterized by complex intrinsic impedance, η.

a) Find $\mathbf{H}_s$: With the wave propagating in the forward z direction, we find:

$$\mathbf{H}_s = \frac{1}{\eta}\left[-18e^{j\phi}\mathbf{a}_x + 15\mathbf{a}_y\right]e^{-j\beta z} \text{ A/m}$$

b) Determine the average power density in W/m^2: We find

$$P_{z,avg} = \frac{1}{2}\text{Re}\left\{\mathbf{E}_s \times \mathbf{H}_s^*\right\} = \frac{1}{2}\text{Re}\left\{\frac{(15)^2}{\eta^*} + \frac{(18)^2}{\eta^*}\right\} = 275\,\text{Re}\left\{\frac{1}{\eta^*}\right\} \text{ W/m}^2$$

11.34. Given the general elliptically-polarized wave as per Eq. (73):

$$\mathbf{E}_s = [E_{x0}\mathbf{a}_x + E_{y0}e^{j\phi}\mathbf{a}_y]e^{-j\beta z}$$

a) Show, using methods similar to those of Example 11.7, that a linearly polarized wave results when superimposing the given field and a phase-shifted field of the form:

$$\mathbf{E}_s = [E_{x0}\mathbf{a}_x + E_{y0}e^{-j\phi}\mathbf{a}_y]e^{-j\beta z}e^{j\delta}$$

where δ is a constant: Adding the two fields gives

$$\mathbf{E}_{s,tot} = \left[E_{x0}\left(1 + e^{j\delta}\right)\mathbf{a}_x + E_{y0}\left(e^{j\phi} + e^{-j\phi}e^{j\delta}\right)\mathbf{a}_y\right]e^{-j\beta z}$$

$$= \left[E_{x0}e^{j\delta/2}\underbrace{\left(e^{-j\delta/2} + e^{j\delta/2}\right)}_{2\cos(\delta/2)}\mathbf{a}_x + E_{y0}e^{j\delta/2}\underbrace{\left(e^{-j\delta/2}e^{j\phi} + e^{-j\phi}e^{j\delta/2}\right)}_{2\cos(\phi-\delta/2)}\mathbf{a}_y\right]e^{-j\beta z}$$

This simplifies to $\mathbf{E}_{s,tot} = 2\left[E_{x0}\cos(\delta/2)\mathbf{a}_x + E_{y0}\cos(\phi - \delta/2)\mathbf{a}_y\right]e^{j\delta/2}e^{-j\beta z}$, which is linearly polarized.

b) Find δ in terms of ϕ such that the resultant wave is polarized along x: By inspecting the part *a* result, we achieve a zero y component when $2\phi - \delta = \pi$ (or odd multiples of π).

CHAPTER 12

12.1. A uniform plane wave in air, $E_{x1}^+ = E_{x10}^+ \cos(10^{10}t - \beta z)$ V/m, is normally-incident on a copper surface at $z = 0$. What percentage of the incident power density is transmitted into the copper? We need to find the reflection coefficient. The intrinsic impedance of copper (a good conductor) is

$$\eta_c = \sqrt{\frac{j\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\sqrt{\frac{10^{10}(4\pi \times 10^7)}{2(5.8 \times 10^7)}} = (1+j)(.0104)$$

Note that the accuracy here is questionable, since we know the conductivity to only two significant figures. We nevertheless proceed: Using $\eta_0 = 376.7288$ ohms, we write

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{.0104 - 376.7288 + j.0104}{.0104 + 376.7288 + j.0104} = -.9999 + j.0001$$

Now $|\Gamma|^2 = .9999$, and so the transmitted power fraction is $1 - |\Gamma|^2 = .0001$, or about 0.01% is transmitted.

12.2. The plane $y = 0$ defines the boundary between two different dielectrics. For $y < 0$, $\epsilon'_{R1} = 1$, $\mu_1 = \mu_0$, and $\epsilon''_{R1} = 0$; and for $y > 0$, $\epsilon'_{R2} = 5$, $\mu_2 = \mu_0$, and $\epsilon''_{R2} = 0$. Let $E_{z1}^+ = 150\cos(\omega t - 8y)$ V/m, and find

a) ω: Have $\beta = 8 = \omega/c \Rightarrow \omega = 8c = \underline{2.4 \times 10^9 \text{ sec}^{-1}}$.

b) $\mathbf{H}_1^+$: With E in the z direction, and propagation in the forward y direction, H will lie in the positive x direction, and its amplitude will be $H_x = E_y/\eta_0$ in region 1.
Thus $\mathbf{H}_1^+ = (150/\eta_0)\cos(\omega t - 8y)\mathbf{a}_x = \underline{0.40\cos(2.4 \times 10^9 t - 8y)\mathbf{a}_x \text{ A/m}}$.

c) $\mathbf{H}_1^-$: First,

$$E_{z1}^- = \Gamma E_{z1}^+ = \frac{\eta_0/\sqrt{5} - \eta_0/1}{\eta_0/\sqrt{5} + \eta_0/1} = \frac{1-\sqrt{5}}{1+\sqrt{5}}E_{z1}^+ = -0.38E_{z1}^+$$

Then

$$H_{x1}^- = +(0.38/\eta_0)E_{z1}^+ = \frac{0.38(150)}{377}\cos(\omega t + 8y)$$

So finally, $\mathbf{H}_{x1}^- = \underline{0.15\cos(2.4 \times 10^9 t + 8y)\mathbf{a}_x \text{ A/m}}$.

12.3. A uniform plane wave in region 1 is normally-incident on the planar boundary separating regions 1 and 2. If $\epsilon''_1 = \epsilon''_2 = 0$, while $\epsilon'_{R1} = \mu_{R1}^3$ and $\epsilon'_{R2} = \mu_{R2}^3$, find the ratio $\epsilon'_{R2}/\epsilon'_{R1}$ if 20% of the energy in the incident wave is reflected at the boundary. There are two possible answers. First, since $|\Gamma|^2 = .20$, and since both permittivities and permeabilities are real, $\Gamma = \pm 0.447$. we then set up

$$\Gamma = \pm 0.447 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0\sqrt{(\mu_{R2}/\epsilon'_{R2})} - \eta_0\sqrt{(\mu_{R1}/\epsilon'_{R1})}}{\eta_0\sqrt{(\mu_{R2}/\epsilon'_{R2})} + \eta_0\sqrt{(\mu_{R1}/\epsilon'_{R1})}}$$

$$= \frac{\sqrt{(\mu_{R2}/\mu_{R2}^3)} - \sqrt{(\mu_{R1}/\mu_{R1}^3)}}{\sqrt{(\mu_{R2}/\mu_{R2}^3)} + \sqrt{(\mu_{R1}/\mu_{R1}^3)}} = \frac{\mu_{R1} - \mu_{R2}}{\mu_{R1} + \mu_{R2}}$$

12.3. (continued) Therefore

$$\frac{\mu_{R2}}{\mu_{R1}} = \frac{1 \mp 0.447}{1 \pm 0.447} = (0.382, 2.62) \;\Rightarrow\; \frac{\epsilon'_{R2}}{\epsilon'_{R1}} = \left(\frac{\mu_{R2}}{\mu_{R1}}\right)^3 = \underline{(0.056, 17.9)}$$

12.4. The magnetic field intensity in a region where $\epsilon'' = 0$ is given as $\mathbf{H} = 5\cos\omega t\cos\beta z\,\mathbf{a}_y$ A/m, where $\omega = 5$ Grad/s and $\beta = 30$ rad/m. If the amplitude of the associated electric field intensity is 2kV/m, find

a) μ and ϵ' for the medium: In phasor form, the magnetic field is $H_{ys} = H_0 e^{-j\beta z} + H_0 e^{+\beta z} = 5\cos\beta z \;\Rightarrow\; H_0 = 2.5$. The electric field will be x directed, and is $E_{xs} = \eta(2.5)e^{-j\beta z} - \eta(2.5)e^{+j\beta z} = (2j)\eta(2.5)\sin\beta z$. Given the electric field amplitude of 2 kV/m, we write $2\times 10^3 = 5\eta$, or $\eta = 400\,\Omega$. Now $\eta = 400 = \eta_0\sqrt{\mu_r/\epsilon'_R}$ and we also have $\beta = 30 = (\omega/c)\sqrt{\mu_R\epsilon'_R}$. We solve these two equations simultaneously for μ_R and ϵ'_R to find $\mu_R = 1.91$ and $\epsilon'_R = 1.70$. Therefore $\mu = 1.91\times 4\pi\times 10^{-7} = \underline{2.40\,\mu\text{H/m}}$ and $\epsilon' = 1.70\times 8.854\times 10^{-12} = \underline{15.1\text{ pF/m}}$.

b) $\mathbf{E}$: From part a, electric field in phasor form is $E_{xs} = j2\sin\beta z$ kV/m, and so, in real form: $\mathbf{E}(z,t) = \text{Re}(E_{xs}e^{j\omega t})\mathbf{a}_x = \underline{2\sin\beta z\sin\omega t\,\mathbf{a}_x\text{ kV/m}}$ with ω and β as given.

12.5. The region $z < 0$ is characterized by $\epsilon'_R = \mu_R = 1$ and $\epsilon''_R = 0$. The total $\mathbf{E}$ field here is given as the sum of the two uniform plane waves, $\mathbf{E}_s = 150e^{-j10z}\,\mathbf{a}_x + (50\angle 20^\circ)e^{j10z}\,\mathbf{a}_x$ V/m.

a) What is the operating frequency? In free space, $\beta = k_0 = 10 = \omega/c = \omega/3\times 10^8$. Thus, $\omega = 3\times 10^9\text{ s}^{-1}$, or $f = \omega/2\pi = \underline{4.7\times 10^8\text{ Hz}}$.

b) Specify the intrinsic impedance of the region $z > 0$ that would provide the appropriate reflected wave: Use

$$\Gamma = \frac{E_r}{E_{inc}} = \frac{50e^{j20^\circ}}{150} = \frac{1}{3}e^{j20^\circ} = 0.31 + j0.11 = \frac{\eta - \eta_0}{\eta + \eta_0}$$

Now

$$\eta = \eta_0\left(\frac{1+\Gamma}{1-\Gamma}\right) = 377\left(\frac{1+0.31+j0.11}{1-0.31-j0.31}\right) = \underline{691 + j177\ \Omega}$$

c) At what value of z ($-10\text{ cm} < z < 0$) is the total electric field intensity a maximum amplitude? We found the phase of the reflection coefficient to be $\phi = 20^\circ = .349$rad, and we use

$$z_{max} = \frac{-\phi}{2\beta} = \frac{-.349}{20} = -0.017\text{ m} = \underline{-1.7\text{ cm}}$$

12.6. Region 1, $z < 0$, and region 2, $z > 0$, are described by the following parameters: $\epsilon'_1 = 100$ pF/m, $\mu_1 = 25\ \mu$H/m, $\epsilon''_1 = 0$, $\epsilon'_2 = 200$ pF/m, $\mu_2 = 50\ \mu$H/m, and $\epsilon''_2/\epsilon'_2 = 0.5$. If $\mathbf{E}_1^+ = 600e^{-\alpha_1 z}\cos(5\times 10^{10}t - \beta_1 z)\mathbf{a}_x$ V/m, find:

a) α_1: From Eq. (35), Chapter 11, we note that since $\epsilon''_1 = 0$, it follows that $\alpha_1 = \underline{0}$.

b) β_1: $\beta_1 = \omega\sqrt{\mu_1\epsilon'_1} = (5\times 10^{10})\sqrt{(25\times 10^{-6})(100\times 10^{-12})} = \underline{2.50\times 10^3\text{ rad/m}}$.

c) $\mathbf{E}_{s1}^+ = \underline{600e^{-j2.50\times 10^3 z}\mathbf{a}_x\text{ V/m}}$.

d) $\mathbf{E}_{s1}^-$: To find this, we need to evaluate the reflection coefficient, which means that we first need the two intrinsic impedances. First, $\eta_1 = \sqrt{\mu_1/\epsilon'_1} = \sqrt{(25\times 10^{-6})/(100\times 10^{-12})} = 500$.

12.6d) (continued) Next, using Eq. (39), Chapter 11,

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon'_2}} \frac{1}{\sqrt{1 - j(\epsilon''_2/\epsilon'_2)}} = \sqrt{\frac{50 \times 10^{-6}}{2 \times 10^{-10}}} \frac{1}{\sqrt{1 - j0.5}} = 460 + j109$$

Then

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{460 + j109 - 500}{460 + j109 + 500} = -2.83 \times 10^{-2} + j1.16 \times 10^{-1} = 0.120e^{j104^\circ}$$

Now we multiply $\mathbf{E}^+_{s1}$ by Γ and reverse the propagation direction to obtain

$$\mathbf{E}^-_{s1} = \underline{71.8e^{j104^\circ} e^{j2.5 \times 10^3 z} \text{ V/m}}$$

e) $\mathbf{E}^+_{s2}$: This wave will experience loss in region 2, along with a different phase constant. We need to evaluate α_2 and β_2. First, using Eq. (35), Chapter 11,

$$\alpha_2 = \omega\sqrt{\frac{\mu_2\epsilon'_2}{2}} \left[\sqrt{1 + \left(\frac{\epsilon''_2}{\epsilon'_2}\right)^2} - 1\right]^{1/2}$$

$$= (5 \times 10^{10})\sqrt{\frac{(50 \times 10^6)(200 \times 10^{-12})}{2}} \left[\sqrt{1 + (0.5)^2} - 1\right]^{1/2} = 1.21 \times 10^3 \text{ Np/m}$$

Then, using Eq. (36), Chapter 11,

$$\beta_2 = \omega\sqrt{\frac{\mu_2\epsilon'_2}{2}} \left[\sqrt{1 + \left(\frac{\epsilon''_2}{\epsilon'_2}\right)^2} + 1\right]^{1/2} = 5.15 \times 10^3 \text{ rad/m}$$

Then, the transmission coefficient will be

$$\tau = 1 + \Gamma = 1 - 2.83 \times 10^{-2} + j1.16 \times 10^{-1} = 0.972e^{j7^\circ}$$

The complex amplitude of $\mathbf{E}^+_{s2}$ is then found by multiplying the amplitude of $\mathbf{E}^+_{s1}$ by τ. The field in region 2 is then constructed by using the resulting amplitude, along with the attenuation and phase constants that are appropriate for region 2. The result is

$$\mathbf{E}^+_{s2} = \underline{587e^{-1.21 \times 10^3 z} e^{j7^\circ} e^{-j5.15 \times 10^3 z} \text{ V/m}}$$

12.7. The semi-infinite regions $z < 0$ and $z > 1$ m are free space. For $0 < z < 1$ m, $\epsilon'_R = 4$, $\mu_R = 1$, and $\epsilon''_R = 0$. A uniform plane wave with $\omega = 4 \times 10^8$ rad/s is travelling in the $\mathbf{a}_z$ direction toward the interface at $z = 0$.

a) Find the standing wave ratio in each of the three regions: First we find the phase constant in the middle region,

$$\beta_2 = \frac{\omega\sqrt{\epsilon'_R}}{c} = \frac{2(4 \times 10^8)}{3 \times 10^8} = 2.67 \text{ rad/m}$$

12.7a. (continued) Then, with the middle layer thickness of 1 m, $\beta_2 d = 2.67$ rad. Also, the intrinsic impedance of the middle layer is $\eta_2 = \eta_0/\sqrt{\epsilon'_R} = \eta_0/2$. We now find the input impedance:

$$\eta_{in} = \eta_2 \left[\frac{\eta_0 \cos(\beta_2 d) + j\eta_2 \sin(\beta_2 d)}{\eta_2 \cos(\beta_2 d) + j\eta_0 \sin(\beta_2 d)} \right] = \frac{377}{2} \left[\frac{2\cos(2.67) + j\sin(2.67)}{\cos(2.67) + j2\sin(2.67)} \right] = 231 + j141$$

Now, at the first interface,

$$\Gamma_{12} = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} = \frac{231 + j141 - 377}{231 + j141 + 377} = -.176 + j.273 = .325\angle 123^\circ$$

The standing wave ratio measured in region 1 is thus

$$s_1 = \frac{1 + |\Gamma_{12}|}{1 - |\Gamma_{12}|} = \frac{1 + 0.325}{1 - 0.325} = \underline{1.96}$$

In region 2 the standing wave ratio is found by considering the reflection coefficient for waves incident from region 2 on the second interface:

$$\Gamma_{23} = \frac{\eta_0 - \eta_0/2}{\eta_0 + \eta_0/2} = \frac{1 - 1/2}{1 + 1/2} = \frac{1}{3}$$

Then

$$s_2 = \frac{1 + 1/3}{1 - 1/3} = \underline{2}$$

Finally, $s_3 = \underline{1}$, since no reflected waves exist in region 3.

b) Find the location of the maximum $|\mathbf{E}|$ for $z < 0$ that is nearest to $z = 0$. We note that the phase of Γ_{12} is $\phi = 123^\circ = 2.15$ rad. Thus

$$z_{max} = \frac{-\phi}{2\beta} = \frac{-2.15}{2(4/3)} = \underline{-.81\,\text{m}}$$

12.8. A wave starts at point a, propagates 100m through a lossy dielectric for which $\alpha = 0.5$ Np/m, reflects at normal incidence at a boundary at which $\Gamma = 0.3 + j0.4$, and then returns to point a. Calculate the ratio of the final power to the incident power after this round trip: Final power, P_f, and incident power, P_i, are related through

$$P_f = P_i e^{-2\alpha L} |\Gamma|^2 e^{-2\alpha L} \quad \Rightarrow \quad \frac{P_f}{P_i} = |0.3 + j0.4|^2 e^{-2(0.5)100} = \underline{3.5 \times 10^{-88}}(!)$$

Try measuring that.

12.9. Region 1, $z < 0$, and region 2, $z > 0$, are both perfect dielectrics ($\mu = \mu_0$, $\epsilon'' = 0$). A uniform plane wave traveling in the $\mathbf{a}_z$ direction has a radian frequency of 3×10^{10} rad/s. Its wavelengths in the two regions are $\lambda_1 = 5$ cm and $\lambda_2 = 3$ cm. What percentage of the energy incident on the boundary is

a) reflected; We first note that

$$\epsilon'_{R1} = \left(\frac{2\pi c}{\lambda_1 \omega} \right)^2 \quad \text{and} \quad \epsilon'_{R2} = \left(\frac{2\pi c}{\lambda_2 \omega} \right)^2$$

12.9a. (continued) Therefore $\epsilon'_{R1}/\epsilon'_{R2} = (\lambda_2/\lambda_1)^2$. Then with $\mu = \mu_0$ in both regions, we find

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0\sqrt{1/\epsilon'_{R2}} - \eta_0\sqrt{1/\epsilon'_{R1}}}{\eta_0\sqrt{1/\epsilon'_{R2}} + \eta_0\sqrt{1/\epsilon'_{R1}}} = \frac{\sqrt{\epsilon'_{R1}/\epsilon'_{R2}} - 1}{\sqrt{\epsilon'_{R1}/\epsilon'_{R2}} + 1} = \frac{(\lambda_2/\lambda_1) - 1}{(\lambda_2/\lambda_1) + 1}$$
$$= \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1} = \frac{3-5}{3+5} = -\frac{1}{4}$$

The fraction of the incident energy that is reflected is then $|\Gamma|^2 = 1/16 = \underline{6.25 \times 10^{-2}}$.

b) transmitted? We use part a and find the transmitted fraction to be $1 - |\Gamma|^2 = 15/16 = \underline{0.938}$.

c) What is the standing wave ratio in region 1? Use

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/4}{1-1/4} = \frac{5}{3} = \underline{1.67}$$

12.10. In Fig. 12.1, let region 2 be free space, while $\mu_{R1} = 1$, $\epsilon''_{R1} = 0$, and ϵ'_{R1} is unknown. Find $\epsilon'_{R!}$ if

a) the amplitude of $\mathbf{E}_1^-$ is one-half that of $\mathbf{E}_1^+$: Since region 2 is free space, the reflection coefficient is

$$\Gamma = \frac{|\mathbf{E}_1^-|}{|\mathbf{E}_1^+|} = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{\eta_0 - \eta_0/\sqrt{\epsilon'_{R1}}}{\eta_0 + \eta_0/\sqrt{\epsilon'_{R1}}} = \frac{\sqrt{\epsilon'_{R1}} - 1}{\sqrt{\epsilon'_{R1}} + 1} = \frac{1}{2} \quad \Rightarrow \quad \epsilon'_{R1} = \underline{9}$$

.

b) $P^-_{1,avg}$ is one-half of $P^+_{1,avg}$: This time

$$|\Gamma|^2 = \left| \frac{\sqrt{\epsilon'_{R1}} - 1}{\sqrt{\epsilon'_{R1}} + 1} \right|^2 = \frac{1}{2} \quad \Rightarrow \quad \epsilon'_{R1} = \underline{34}$$

c) $|\mathbf{E}_1|_{min}$ is one-half $|\mathbf{E}_1|_{max}$: Use

$$\frac{|\mathbf{E}_1|_{max}}{|\mathbf{E}_1|_{min}} = s = \frac{1+|\Gamma|}{1-|\Gamma|} = 2 \quad \Rightarrow \quad |\Gamma| = \Gamma = \frac{1}{3} = \frac{\sqrt{\epsilon'_{R1}} - 1}{\sqrt{\epsilon'_{R1}} + 1} \quad \Rightarrow \quad \epsilon'_{R1} = \underline{4}$$

12.11. A 150 MHz uniform plane wave in normally-incident from air onto a material whose intrinsic impedance is unknown. Measurements yield a standing wave ratio of 3 and the appearance of an electric field minimum at 0.3 wavelengths in front of the interface. Determine the impedance of the unknown material: First, the field minimum is used to find the phase of the reflection coefficient, where

$$z_{min} = -\frac{1}{2\beta}(\phi + \pi) = -0.3\lambda \quad \Rightarrow \quad \phi = 0.2\pi$$

where $\beta = 2\pi/\lambda$ has been used. Next,

$$|\Gamma| = \frac{s-1}{s+1} = \frac{3-1}{3+1} = \frac{1}{2}$$

12.11. (continued) So we now have

$$\Gamma = 0.5e^{j0.2\pi} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

We solve for η_u to find

$$\eta_u = \eta_0(1.70 + j1.33) = \underline{641 + j501\ \Omega}$$

12.12. A 50MHz uniform plane wave is normally incident from air onto the surface of a calm ocean. For seawater, $\sigma = 4$ S/m, and $\epsilon'_R = 78$.

a) Determine the fractions of the incident power that are reflected and transmitted: First we find the loss tangent:

$$\frac{\sigma}{\omega\epsilon'} = \frac{4}{2\pi(50 \times 10^6)(78)(8.854 \times 10^{-12})} = 18.4$$

This value is sufficiently greater than 1 to enable seawater to be considered a good conductor at 50MHz. Then, using the approximation (Eq. 65, Chapter 11), the intrinsic impedance is $\eta_s = \sqrt{\pi f \mu/\sigma}(1 + j)$, and the reflection coefficient becomes

$$\Gamma = \frac{\sqrt{\pi f \mu/\sigma}\ (1 + j) - \eta_0}{\sqrt{\pi f \mu/\sigma}\ (1 + j) + \eta_0}$$

where $\sqrt{\pi f \mu/\sigma} = \sqrt{\pi(50 \times 10^6)(4\pi \times 10^{-7})/4} = 7.0$. The fraction of the power reflected is

$$\frac{P_r}{P_i} = |\Gamma|^2 = \frac{[\sqrt{\pi f \mu/\sigma} - \eta_0]^2 + \pi f \mu/\sigma}{[\sqrt{\pi f \mu/\sigma} + \eta_0]^2 + \pi f \mu/\sigma} = \frac{[7.0 - 377]^2 + 49.0}{[7.0 + 377]^2 + 49.0} = \underline{0.93}$$

The transmitted fraction is then

$$\frac{P_t}{P_i} = 1 - |\Gamma|^2 = 1 - 0.93 = \underline{0.07}$$

b) Qualitatively, how will these answers change (if at all) as the frequency is increased? Within the limits of our good conductor approximation (loss tangent greater than about ten), the reflected power fraction, using the formula derived in part *a*, is found to <u>decrease</u> with increasing frequency. The transmitted power fraction thus increases.

12.13. A right-circularly-polarized plane wave is normally incident from air onto a semi-infinite slab of plexiglas ($\epsilon'_R = 3.45$, $\epsilon''_R = 0$). Calculate the fractions of the incident power that are reflected and transmitted. Also, describe the polarizations of the reflected and transmitted waves. First, the impedance of the plexiglas will be $\eta = \eta_0/\sqrt{3.45} = 203\,\Omega$. Then

$$\Gamma = \frac{203 - 377}{203 + 377} = -0.30$$

The reflected power fraction is thus $|\Gamma|^2 = \underline{0.09}$. The total electric field in the plane of the interface must rotate in the same direction as the incident field, in order to continually satisfy the boundary condition of tangential electric field continuity across the interface. Therefore, the reflected wave will have to be <u>left circularly polarized</u> in order to make this happen. The transmitted power fraction is now $1 - |\Gamma|^2 = \underline{0.91}$. The transmitted field will be <u>right circularly polarized</u> (as the incident field) for the same reasons.

12.14. A left-circularly-polarized plane wave is normally-incident onto the surface of a perfect conductor.

a) Construct the superposition of the incident and reflected waves in phasor form: Assume positive z travel for the incident electric field. Then, with reflection coefficient, $\Gamma = -1$, the incident and reflected fields will add to give the total field:

$$\mathbf{E}_{tot} = \mathbf{E}_i + \mathbf{E}_r = E_0(\mathbf{a}_x + j\mathbf{a}_y)e^{-j\beta z} - E_0(\mathbf{a}_x + j\mathbf{a}_y)e^{+j\beta z}$$

$$= E_0 \left[\underbrace{\left(e^{-j\beta z} - e^{j\beta z}\right)}_{-2j\sin(\beta z)} \mathbf{a}_x + j \underbrace{\left(e^{-j\beta z} - e^{j\beta z}\right)}_{-2j\sin(\beta z)} \mathbf{a}_y \right] = \underline{2E_0 \sin(\beta z)\left[\mathbf{a}_y - j\mathbf{a}_x\right]}$$

b) Determine the real instantaneous form of the result of part a:

$$\mathbf{E}(z,t) = \text{Re}\left\{\mathbf{E}_{tot}e^{j\omega t}\right\} = \underline{2E_0\sin(\beta z)\left[\cos(\omega t)\mathbf{a}_y + \sin(\omega t)\mathbf{a}_x\right]}$$

c) Describe the wave that is formed: This is a standing wave exhibiting circular polarization in time. At each location along the z axis, the field vector rotates clockwise in the xy plane, and has amplitude (constant with time) given by $2E_0\sin(\beta z)$.

12.15. Consider these regions in which $\epsilon'' = 0$: region 1, $z < 0$, $\mu_1 = 4\,\mu$H/m and $\epsilon'_1 = 10$ pF/m; region 2, $0 < z < 6$ cm, $\mu_2 = 2\,\mu H/m$, $\epsilon'_2 = 25$ pF/m; region 3, $z > 6$ cm, $\mu_3 = \mu_1$ and $\epsilon'_3 = \epsilon'_1$.

a) What is the lowest frequency at which a uniform plane wave incident from region 1 onto the boundary at $z = 0$ will have no reflection? This frequency gives the condition $\beta_2 d = \pi$, where $d = 6$ cm, and $\beta_2 = \omega\sqrt{\mu_2\epsilon'_2}$ Therefore

$$\beta_2 d = \pi \quad\Rightarrow\quad \omega = \frac{\pi}{(.06)\sqrt{\mu_2\epsilon'_2}} \quad\Rightarrow\quad f = \frac{1}{0.12\sqrt{(2\times10^{-6})(25\times10^{-12})}} = \underline{1.2\text{ GHz}}$$

b) If $f = 50$ MHz, what will the standing wave ratio be in region 1? At the given frequency, $\beta_2 = (2\pi\times5\times10^7)\sqrt{(2\times10^{-6})(25\times10^{-12})} = 2.22$ rad/m. Thus $\beta_2 d = 2.22(.06) = 0.133$. The intrinsic impedance of regions 1 and 3 is $\eta_1 = \eta_3 = \sqrt{(4\times10^{-6})/(10^{-11})} = 632\,\Omega$. The input impedance at the first interface is now

$$\eta_{in} = 283\left[\frac{632\cos(.133) + j283\sin(.133)}{283\cos(.133) + j632\sin(.133)}\right] = 589 - j138 = 605\angle - .23$$

The reflection coefficient is now

$$\Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1} = \frac{589 - j138 - 632}{589 - j138 + 632} = .12\angle - 1.7$$

The standing wave ratio is now

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + .12}{1 - .12} = \underline{1.27}$$

12.16. A uniform plane wave in air is normally-incident onto a lossless dielectric plate of thickness $\lambda/8$, and of intrinsic impedance $\eta = 260\ \Omega$. Determine the standing wave ratio in front of the plate. Also find the fraction of the incident power that is transmitted to the other side of the plate: With the a thickness of $\lambda/8$, we have $\beta d = \pi/4$, and so $\cos(\beta d) = \sin(\beta d) = 1\sqrt{2}$. The input impedance thus becomes

$$\eta_{in} = 260\left[\frac{377 + j260}{260 + j377}\right] = 243 - j92\ \Omega$$

12.16. (continued)

The reflection coefficient is then

$$\Gamma = \frac{(243 - j92) - 377}{(243 - j92) + 377} = -0.19 - j0.18 = 0.26\angle - 2.4\text{rad}$$

Therefore

$$s = \frac{1 + .26}{1 - .26} = \underline{1.7} \text{ and } 1 - |\Gamma|^2 = 1 - (.26)^2 = \underline{0.93}$$

12.17. Repeat Problem 12.16 for the cases in which the frequency is

a) doubled: If this is true, then $d = \lambda/4$, and thus $\eta_{in} = (260)^2/377 = 179$. The reflection coefficient becomes

$$\Gamma = \frac{179 - 377}{179 + 377} = -0.36 \quad \Rightarrow \quad s = \frac{1 + .36}{1 - .36} = \underline{2.13}$$

Then $1 - |\Gamma|^2 = 1 - (.36)^2 = \underline{0.87}$.

b) quadrupled: Now, $d = \lambda/2$, and so we have a half-wave section surrounded by air. Transmission will be total, and so $s = \underline{1}$ and $1 - |\Gamma|^2 = \underline{1}$.

12.18. In Fig. 12.6, let $\eta_1 = \eta_3 = 377\Omega$, and $\eta_2 = 0.4\eta_1$. A uniform plane wave is normally incident from the left, as shown. Plot a curve of the standing wave ratio, s, in the region to the left:

a) as a function of l if $f = 2.5$GHz: With $\eta_1 = \eta_3 = \eta_0$ and with $\eta_2 = 0.4\eta_0$, Eq. (41) becomes

$$\eta_{in} = 0.4\eta_0 \left[\frac{\cos(\beta l) + j0.4\sin(\beta l)}{0.4\cos(\beta l) + j\sin(\beta l)} \right] \times \left[\frac{0.4\cos(\beta l) - j\sin(\beta l)}{0.4\cos(\beta l) - j\sin(\beta l)} \right]$$
$$= \eta_0 \left[\frac{1 - j1.05\sin(2\beta l)}{\cos^2(\beta l) + 6.25\sin^2(\beta l)} \right]$$

Then $\Gamma = (\eta_{in} - \eta_0)/(\eta_{in} + \eta_0)$, from which we find

$$|\Gamma| = \sqrt{\Gamma\Gamma^*} = \left[\frac{\left[1 - \cos^2(\beta l) - 6.25\sin^2(\beta l)\right]^2 + (1.05)^2\sin^2(2\beta l)}{\left[1 + \cos^2(\beta l) + 6.25\sin^2(\beta l)\right]^2 + (1.05)^2\sin^2(2\beta l)} \right]^{1/2}$$

Then $s = (1 + |\Gamma|)/(1 - |\Gamma|)$. Now for a uniform plane wave, $\beta = \omega\sqrt{\mu\epsilon} = n\omega/c$. Given that $\eta_2 = 0.4\eta_0 = \eta_0/n$, we find $n = 2.5$ (assuming $\mu = \mu_0$). Thus, at 2.5 GHz,

$$\beta l = \frac{n\omega}{c} l = \frac{(2.5)(2\pi)(2.5 \times 10^9)}{3 \times 10^8} l = 12.95\, l \ (l \text{ in m}) = 0.1295\, l \ (l \text{ in cm})$$

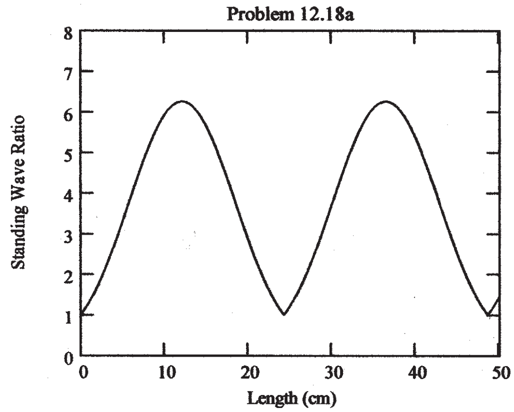
Using this in the expression for $|\Gamma|$, and calculating s as a function of l in cm leads to the first plot shown on the next page.

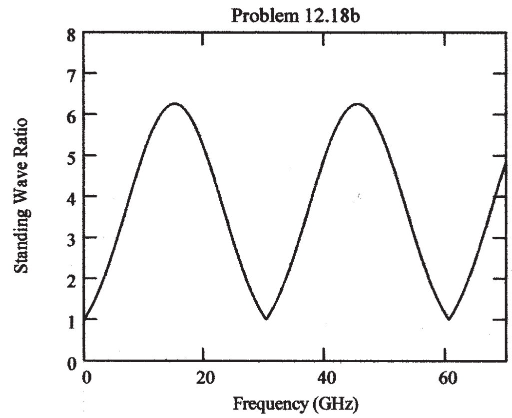
b) as a function of frequency if $l = 2$cm. In this case we use

$$\beta l = \frac{(2.5)(2\pi)(0.02)}{3 \times 10^8} f = 1.04 \times 10^{-10} f \ (f \text{ in Hz}) = 0.104 f \ (f \text{ in GHz})$$

Using this in the expression for $|\Gamma|$, and calculating s as a function of f in GHz leads to the second plot shown on the next page. MathCad was used in both cases.

12.18 (continued) Plots for parts *a* and *b*





12.19. You are given four slabs of lossless dielectric, all with the same intrinsic impedance, η, known to be different from that of free space. The thickness of each slab is $\lambda/4$, where λ is the wavelength as measured in the slab material. The slabs are to be positioned parallel to one another, and the combination lies in the path of a uniform plane wave, normally-incident. The slabs are to be arranged such that the air spaces between them are either zero, one-quarter wavelength, or one-half wavelength in thickness. Specify an arrangement of slabs and air spaces such that

a) the wave is totally transmitted through the stack: In this case, we look for a combination of half-wave sections. Let the inter-slab distances be d_1, d_2, and d_3 (from left to right). Two possibilities are i.) $\underline{d_1 = d_2 = d_3 = 0}$, thus creating a single section of thickness λ, or ii.) $\underline{d_1 = d_3 = 0,\ d_2 = \lambda/2}$, thus yielding two half-wave sections separated by a half-wavelength.

b) the stack presents the highest reflectivity to the incident wave: The best choice here is to make $\underline{d_1 = d_2 = d_3 = \lambda/4}$. Thus every thickness is one-quarter wavelength. The impedances transform as follows: First, the input impedance at the front surface of the last slab (slab 4) is $\eta_{in,1} = \eta^2/\eta_0$. We transform this back to the back surface of slab 3, moving through a distance of $\lambda/4$ in free space: $\eta_{in,2} = \eta_0^2/\eta_{in,1} = \eta_0^3/\eta^2$. We next transform this impedance to the front surface of slab 3, producing $\eta_{in,3} = \eta^2/\eta_{in,2} = \eta^4/\eta_0^3$. We continue in this manner until reaching the front surface of slab 1, where we find $\eta_{in,7} = \eta^8/\eta_0^7$. Assuming $\eta < \eta_0$, the ratio η^n/η_0^{n-1} becomes smaller as n increases (as the number of slabs increases). The reflection coefficient for waves incident on the front slab thus gets close to unity, and approaches 1 as the number of slabs approaches infinity.

12.20. The 50MHz plane wave of Problem 12.12 is incident onto the ocean surface at an angle to the normal of 60°. Determine the fractions of the incident power that are reflected and transmitted for

a) s polarization: To review Problem 12, we first we find the loss tangent:

$$\frac{\sigma}{\omega\epsilon'} = \frac{4}{2\pi(50\times 10^6)(78)(8.854\times 10^{-12})} = 18.4$$

This value is sufficiently greater than 1 to enable seawater to be considered a good conductor at 50MHz. Then, using the approximation (Eq. 65, Chapter 11), and with $\mu = \mu_0$, the intrinsic impedance is $\eta_s = \sqrt{\pi f\mu/\sigma}(1+j) = 7.0(1+j)$.

12.20a. (continued)

Next we need the angle of refraction, which means that we need to know the refractive index of seawater at 50MHz. For a uniform plane wave in a good conductor, the phase constant is

$$\beta = \frac{n_{sea}\,\omega}{c} \doteq \sqrt{\pi f \mu \sigma} \quad \Rightarrow \quad n_{sea} \doteq c\sqrt{\frac{\mu\sigma}{4\pi f}} = 26.8$$

Then, using Snell's law, the angle of refraction is found:

$$\sin\theta_2 = \frac{n_{sea}}{n_1}\sin\theta_1 = 26.8\sin(60^\circ) \quad \Rightarrow \quad \theta_2 = 1.9^\circ$$

This angle is small enough so that $\cos\theta_2 \doteq 1$. Therefore, for s polarization,

$$\Gamma_s \doteq \frac{\eta_{s2} - \eta_{s1}}{\eta_{s2} + \eta_{s1}} = \frac{7.0(1+j) - 377/\cos 60^\circ}{7.0(1+j) + 377/\cos 60^\circ} = -0.98 + j0.018 = 0.98\angle 179^\circ$$

The fraction of the power reflected is now $|\Gamma_s|^2 = \underline{0.96}$. The fraction transmitted is then $\underline{0.04}$.

b) p polarization: Again, with the refracted angle close to zero, the relection coefficient for p polarization is

$$\Gamma_p \doteq \frac{\eta_{p2} - \eta_{p1}}{\eta_{p2} + \eta_{p1}} = \frac{7.0(1+j) - 377\cos 60^\circ}{7.0(1+j) + 377\cos 60^\circ} = -0.93 + j0.069 = 0.93\angle 176^\circ$$

The fraction of the power reflected is now $|\Gamma_p|^2 = \underline{0.86}$. The fraction transmitted is then $\underline{0.14}$.

12.21. A right-circularly polarized plane wave in air is incident at Brewster's angle onto a semi-infinite slab of plexiglas ($\epsilon'_R = 3.45$, $\epsilon''_R = 0$, $\mu = \mu_0$).

a) Determine the fractions of the incident power that are reflected and transmitted: In plexiglas, Brewster's angle is $\theta_B = \theta_1 = \tan^{-1}(\epsilon'_{R2}/\epsilon'_{R1}) = \tan^{-1}(\sqrt{3.45}) = 61.7^\circ$. Then the angle of refraction is $\theta_2 = 90^\circ - \theta_B$ (see Example 12.9), or $\theta_2 = 28.3^\circ$. With incidence at Brewster's angle, all p-polarized power will be transmitted — only s-polarized power will be reflected. This is found through

$$\Gamma_s = \frac{\eta_{2s} - \eta_{1s}}{\eta_{2s} + \eta_{1s}} = \frac{.614\eta_0 - 2.11\eta_0}{.614\eta_0 + 2.11\eta_0} = -0.549$$

where $\eta_{1s} = \eta_1\sec\theta_1 = \eta_0\sec(61.7^\circ) = 2.11\eta_0$, and $\eta_{2s} = \eta_2\sec\theta_2 = (\eta_0/\sqrt{3.45})\sec(28.3^\circ) = 0.614\eta_0$. Now, the reflected power fraction is $|\Gamma|^2 = (-.549)^2 = .302$. Since the wave is circularly-polarized, the s-polarized component represents one-half the total incident wave power, and so the fraction of the *total* power that is reflected is $.302/2 = 0.15$, or $\underline{15\%}$. The fraction of the incident power that is transmitted is then the remainder, or $\underline{85\%}$.

b) Describe the polarizations of the reflected and transmitted waves: Since all the p-polarized component is transmitted, the reflected wave will be entirely s-polarized (linear). The transmitted wave, while having all the incident p-polarized power, will have a reduced s-component, and so this wave will be right-elliptically polarized.

12.22. A dielectric waveguide is shown in Fig. 12.18 with refractive indices as labeled. Incident light enters the guide at angle ϕ from the front surface normal as shown. Once inside, the light totally reflects at the upper $n_1 - n_2$ interface, where $n_1 > n_2$. All subsequent reflections from the upper an lower boundaries will be total as well, and so the light is confined to the guide. Express, in terms of n_1 and n_2, the maximum value of ϕ such that total confinement will occur, with $n_0 = 1$. The quantity $\sin\phi$ is known as the *numerical aperture* of the guide.

From the illustration we see that ϕ_1 maximizes when θ_1 is at its minimum value. This minimum will be the critical angle for the $n_1 - n_2$ interface, where $\sin\theta_c = \sin\theta_1 = n_2/n_1$. Let the refracted angle to the right of the vertical interface (not shown) be ϕ_2, where $n_0 \sin\phi_1 = n_1 \sin\phi_2$. Then we see that $\phi_2 + \theta_1 = 90^\circ$, and so $\sin\theta_1 = \cos\phi_2$. Now, the numerical aperture becomes

$$\sin\phi_{1max} = \frac{n_1}{n_0}\sin\phi_2 = n_1\cos\theta_1 = n_1\sqrt{1-\sin^2\theta_1} = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}$$

Finally, $\phi_{1max} = \underline{\sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)}$ is the numerical aperture angle.

12.23. Suppose that ϕ_1 in Fig. 12.18 is Brewster's angle, and that θ_1 is the critical angle. Find n_0 in terms of n_1 and n_2: With the incoming ray at Brewster's angle, the refracted angle of this ray (measured from the inside normal to the front surface) will be $90^\circ - \phi_1$. Therefore, $\phi_1 = \theta_1$, and thus $\sin\phi_1 = \sin\theta_1$. Thus

$$\sin\phi_1 = \frac{n_1}{\sqrt{n_0^2 + n_1^2}} = \sin\theta_1 = \frac{n_2}{n_1} \quad\Rightarrow\quad n_0 = \underline{(n_1/n_2)\sqrt{n_1^2 - n_2^2}}$$

Alternatively, we could have used the result of Problem 12.22, in which it was found that $\sin\phi_1 = (1/n_0)\sqrt{n_1^2 - n_2^2}$, which we then set equal to $\sin\theta_1 = n_2/n_1$ to get the same result.

12.24. A *Brewster prism* is designed to pass p-polarized light without any reflective loss. The prism of Fig. 12.19 is made of glass ($n = 1.45$), and is in air. Considering the light path shown, determine the apex angle, α: With entrance and exit rays at Brewster's angle (to eliminate reflective loss), the interior ray must be horizontal, or parallel to the bottom surface of the prism. From the geometry, the angle between the interior ray and the normal to the prism surfaces that it intersects is $\alpha/2$. Since this angle is also Brewster's angle, we may write:

$$\alpha = 2\sin^{-1}\left(\frac{1}{\sqrt{1+n^2}}\right) = 2\sin^{-1}\left(\frac{1}{\sqrt{1+(1.45)^2}}\right) = 1.21\,\text{rad} = \underline{69.2^\circ}$$

12.25. In the Brewster prism of Fig. 12.19, determine for s-polarized light the fraction of the incident power that is transmitted through the prism: We use $\Gamma_s = (\eta_{s2} - \eta_{s1})/(\eta_{s2} + \eta_{s1})$, where

$$\eta_{s2} = \frac{\eta_2}{\cos(\theta_{B2})} = \frac{\eta_2}{n/\sqrt{1+n^2}} = \frac{\eta_0}{n^2}\sqrt{1+n^2}$$

and

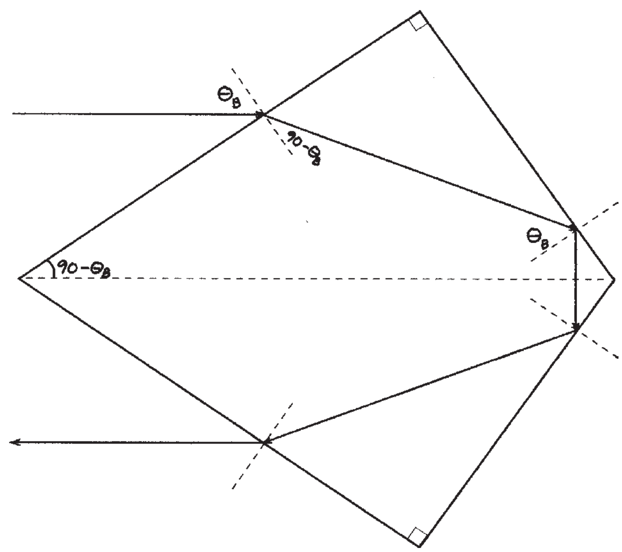
$$\eta_{s1} = \frac{\eta_1}{\cos(\theta_{B1})} = \frac{\eta_1}{1/\sqrt{1+n^2}} = \eta_0\sqrt{1+n^2}$$

12.25. (continued) Thus, at the first interface, $\Gamma = (1-n^2)/(1+n^2)$. At the second interface, Γ will be equal but of opposite sign to the above value. The power transmission coefficient through each interface is $1 - |\Gamma|^2$, so that for both interfaces, we have, with $n = 1.45$:

$$\frac{P_{tr}}{P_{inc}} = \left(1 - |\Gamma|^2\right)^2 = \left[1 - \left(\frac{n^2-1}{n^2+1}\right)^2\right]^2 = \underline{0.76}$$

12.26. Show how a single block of glass can be used to turn a p-polarized beam of iight through 180°, with the light suffering, in principle, zero reflective loss. The light is incident from air, and the returning beam (also in air) may be displaced sideways from the incident beam. Specify all pertinent angles and use $n = 1.45$ for glass. More than one design is possible here.

The prism below is designed such that light enters at Brewster's angle, and once inside, is turned around using total reflection. Using the result of Example 12.9, we find that with glass, $\theta_B = 55.4°$, which, by the geometry, is also the incident angle for total reflection at the back of the prism. For this to work, the Brewster angle must be greater than or equal to the critical angle. This is in fact the case, since $\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1/1.45) = 43.6°$.



12.27. Using Eq. (59) in Chapter 11 as a starting point, determine the ratio of the group and phase velocities of an electromagnetic wave in a good conductor. Assume conductivity does not vary with frequency: In a good conductor:

$$\beta = \sqrt{\pi f \mu\sigma} = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad \rightarrow \qquad \frac{d\beta}{d\omega} = \frac{1}{2}\left[\frac{\omega\mu\sigma}{2}\right]^{-1/2}\frac{\mu\sigma}{2}$$

Thus

$$\frac{d\omega}{d\beta} = \left(\frac{d\beta}{d\omega}\right)^{-1} = 2\sqrt{\frac{2\omega}{\mu\sigma}} = v_g \qquad \text{and} \qquad v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega\mu\sigma/2}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Therefore $v_g/v_p = \underline{2}$.

12.28. Over a certain frequency range, the refractive index of a certain material varies approximately linearly with frequency: $n(\omega) \doteq n_a + n_b(\omega - \omega_a)$, where n_a, n_b, and ω_a are constants. Using $\beta = n\omega/c$:

a) determine the group velocity as a function (or perhaps not a function) of frequency: $v_g = (d\beta/d\omega)^{-1}$, where

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega}\left[\frac{n_a\omega}{c} + \frac{n_b(\omega - \omega_a)\omega}{c}\right] = \frac{1}{c}\left[n_a + n_b(2\omega - \omega_a)\right]$$

so that

$$v_g(\omega) = \underline{c\left[n_a + n_b(2\omega - \omega_a)\right]^{-1}}$$

b) determine the group dispersion parameter, β_2:

$$\beta_2 = \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega_0} = \left.\frac{d}{d\omega}\frac{1}{c}\left[n_a + n_b(2\omega - \omega_a)\right]\right|_{\omega_0} = \underline{2n_b/c}$$

c) Discuss the implications of these results, if any, on pulse broadening: The point of this problem was to show that higher order terms (involving $d^3\beta/d\omega^3$ and higher) in the Taylor series expansion, Eq. (89), do not exist if the refractive index varies linearly with ω. These higher order terms would be necessary in cases involving pulses of exremely large bandwidth, or in media exhibiting complicated variations in their ω-β curves over relatively small frequency ranges. With $d^2\beta/d\omega^2$ constant, the three-term Taylor expansion of Eq. (89) describes the phase constant of this medium exactly. The pulse will broaden and will acquire a frequency sweep (chirp) that is precisely linear with time. Additionally, a pulse of a given bandwidth will broaden by the same amount, regardless of what carrier frequency is used.

12.29. A $T = 5$ ps transform-limited pulse propagates in a dispersive channel for which $\beta_2 = 10$ ps^2/km. Over what distance will the pulse spread to twice its initial width? After propagation, the width is $T' = \sqrt{T^2 + (\Delta\tau)^2} = 2T$. Thus $\Delta\tau = \sqrt{3}T$, where $\Delta\tau = \beta_2 z/T$. Therefore

$$\frac{\beta_2 z}{T} = \sqrt{3}T \text{ or } z = \frac{\sqrt{3}T^2}{\beta_2} = \frac{\sqrt{3}(5\,\text{ps})^2}{10\,\text{ps}^2/\text{km}} = \underline{4.3\text{ km}}$$

12.30. A $T = 20$ ps transform-limited pulse propagates through 10 km of a dispersive channel for which $\beta_2 = 12$ ps^2/km. The pulse then propagates through a second 10 km channel for which $\beta_2 = -12$ ps^2/km. Describe the pulse at the output of the second channel and give a physical explanation for what happened.

Our theory of pulse spreading will allow for changes in β_2 down the length of the channel. In fact, we may write in general:

$$\Delta\tau = \frac{1}{T}\int_0^L \beta_2(z)\,dz$$

Having β_2 change sign at the midpoint, yields a zero $\Delta\tau$, and so the pulse emerges from the output unchanged! Physically, the pulse acquires a positive linear chirp (frequency increases with time over the pulse envelope) during the first half of the channel. When β_2 switches sign, the pulse begins to acquire a negative chirp in the second half, which, over an equal distance, will completely eliminate the chirp acquired during the first half. The pulse, if originally transform-limited at input, will emerge, again transform-limited, at its original width. More generally, complete *dispersion compensation* is achieved using a two-segment channel when $\beta_2 L = -\beta_2' L'$, assuming dispersion terms of higher order than β_2 do not exist.